Oligopoly Models and Information Transmission.

by

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Declaration

Chapter VI represents joint work with Norman J. Ireland, University of Warwick, of which my contribution should be considered fifty percent.

Summary

The thesis contains 5 independent papers together with an introduction and a general conclusion. All five papers consider private information in simple oligopoly models with linear demand and cost functions. The problem to be analysed is the extend to which private information is transmitted between firms and the concequences thereof. In principle the transmission (or dissemination) can take place voluntarily or involuntarily. In the case of voluntary information transmission (or sharing) we assume that this is done honestly. One of the main results in this strand of the literature is that firms have no incentives to share information unless they can collude over strategies. In chapter II and III we show that this conclusion is not generally true.

In chapter II we consider the incentive for risk-averse firms to share their private information. We show that the assumption of risk-aversion in some cases reverse the conclusion in the literature. In chapter III we show that there are cases in which private information and the sharing thereof within a collusive arrangement prove detrimental to the size of a stable collusive arrangement. Thus in some cases private information imply a disincentive to collude.

Chapter IV and V looks at the effect of uncertainty and private information on a two-stage duopoly model in which firms first choose capacity, then compete over prices. In chapter IV we show that no pure strategy equilibrium exists regardless of whether uncertainty is resolved before or after capacity is chosen. A mixed strategy equilibrium is shown to exist, and the equilibrium is worked out for a specific distribution of the random variable. In chapter V we modify the equilibrium concept by imposing a no-mill-price-undercutting rule. We shown that if firms' capacities differ, the firm with the highest capacity endogeneously sets the higher price. Examples of private asymmetric information are considered and the main finding from the examples are that there are cases where neither firm wants to share the information of the best informed.

Chapter VI which is joint work with Norman J. Ireland considers involuntary information transmission via output plans. This allows us to rationalise positive consistent conjectures in a simple oligopoly model.

General for all the models considered is that the results not only differ from those found under certainty, but also that the results are possibly non-robust, especially with regards to changes in information structures and functional forms.

CHAPTER I

INTRODUCTION.

The introduction of uncertainty into models of industrial organisation is becomming more and more widespread, no doubt due to its obvious relevance. Not only are firms subject to an uncertain environment. They base their decisions in part on information which is private or particular to that firm. Firms may know their own local demand, local wages, their own technology better that they know that of their competitors.

The effect of private information has been analysed extensively in models with many agents, see Grossman(1981) for an overview. The analysis of cases where firms are few has only recently been undertaken, no doubt facilitated by the more and more readily available techniques for solving these models under various informational assumptions. In this respect, the emergence of the Rational Expectations solution techniques has been important. In deterministic models of imperfect competition we normally assume that firms take into consideration the reactions of other firms when choosing their own strategy. Under uncertainty, and especially when firms have private information, the assumption that firms use their information consistently when forming expectations is natural.

What makes the oligopoly models difficult to solve when firms have private information is that a firm must have a conjecture about both how the other firms react to its strategies and how they react to their own private information. This is brought out clearly in the models on learning, (e.g. Cyert and DeGroot(1974), Kirman(1975,1983)), where firms are uncertain about their residual demand function. Their private information is the historical data on their own output level and price. The main result from their models is that, if firms start out with a misspecified model of their residual demand they may never learn the true model, but they may converge to a self-fulfilling belief which differs from the true model.

Other oligopoly models with uncertainty have been analysed by Basar and Ho(1974), Levine and Ponssard(1979), Ponssard(1979) and Sakai(1984,1985).

In a duopoly model with linear demand, having random intercept, Basar and Ho(1974) shows the existence and uniqueness of affine equilibrium strategies when firms observe a linear signal on a random variable. Levine and Ponssard(1979) considers the value of information in two-player zero-sum games. They show that information has a value to the player. Ponssard(1979) considers a n-firm oligopoly model where firms producing a homogeneous good are faced with a linear inverse demand curve with random intercept. There are two types of firms, informed and uninformed. A Cournot-Nash equilibrium is shown to exist with expected profits of the informed being an increasing function of the number of informed firms. Information is showed to have a positive value. Sakai(1985) considers a simple duopoly model to evaluate information in the case where cost functions are subject to uncertainty. Firms can gather information about its own costs, the costs of its opponent, the costs of both or nothing at all. Firms are assumed risk-neutral Cournot players. Whether or not information has a value depends on the size of the variance of costs and their correlation. If costs are positively correlated more information may be harmfull to the firms. Sakai(1984) considers a model like the above, only now one of the players is assumed to be a Stackelberg leader. Again information is shown to have a value. In deriving the results, it is assumed that the follower does not learn the information of the leader. This should be kept in mind when interpreting the results.

Private information may get disseminated via firm behaviour. This has been considered within models of competition where dissemination typically takes place via the price system (for a survey of dissemination of information in competitive financial markets, see Andersen(1985)). Dissemination of information is more likely to take place in industries with few firms because these firms exercise a greater influence on each other and hence have a greater incentive to monitor each others actions.

It is this aspect of oligopoly models with uncertainty which is considered in this thesis, i.e. the extend to which private information is transmitted between firms in industries where the number of firms is small. Private information can in principle be transmitted in two ways, voluntarily and involuntarily.

By voluntary information transmission we understand situations where one firm, either directly or through a third party transmit all or part of its information to another firm. It is assumed throughout that the information transmitted this way should be true. One possible interpretation is that firms submit their information to a third party - an auditor - who discards all untrue information, or who can punish a lying firm sufficiently hard to ensure truth-telling. The models which have sofar been analysed in the literature, and the models which will be analysed in the following are all essentially one-period models (as goods are only traded once, the two-stage models in chapters IV and V can be seen as covering only one, albeit long, period in time). These models are not well suited to analyse deceptive information transmission because all firms have an incentive to overstate bad news and understate good news. Hence they will never be believed or the extend of deception can be unravelled by the others. We are thus not concerned with information as a mean to decieve others. Rather we try to see to what extend truthfull information is beneficial. Another argument in favour of doing this is that, if information transmission was beneficial conditional on the information being true, then surely these firms would set up a body or an institution to ensure this. Further, often such third parties do exists (e.g. trade associations and statistical offices). Hence what may seem as involuntary information transmission may be a way of implementing truthful information transmission, necessary because, although voluntary information transmission was beneficial in the first place, truthtelling cannot be insured. Some of the services of the statistical office could possibly be interpreted in this way.

One might ask why a firm would want to share its private information with its competitors. Firstly it allows firms to follow demand more closely as well as allowing for more efficient production. Secondly, it may facilitate informal collusive arrangements as these become easier to monitor. One might expect that information transmission is more likely to be preferred if the private information is about demand factors (at times referred to as public variables, as they affect all firms directly) exactly because this directly enables firms to follow demand more closely and to choose a more efficient production process. On the other hand firms may be less willing if the private information is about some aspect of the firms cost or technology (often referred to as private variables as they only affect others indirectly), because firms might give away information which would put them at a strong disadvantage in e.g. price wars. A report by the Danish monopoly authorities has more than once been suppressed at the wish of the firms in question, because it contained too detailed information on technology and cost-structures in an industry.

As it turns out the results in the literature differs markedly from this. A series of papers: Clarke(1983a,b), Gal-Or(1985a,1986), Li(1986), Novshek and Sonnenshein(1982) and Vives(1984) have considered the incentives to share information. The standard assumptions of these models, which will also to a large extend be made use of in the thesis, are: (a) Linear demand function. (b) Constant marginal costs. (c) Either random demand intercept or random marginal costs about which each firm recieve a signal. (d) The expectations of the random variable conditional on the signal is linear in the signal. (e) The decision to share is made before recieving the signal and firms can commit themselves to share honestly.

In a model with normal distributed random intercept and normal distributed signals, Novshek and Sonnenshein(1982) found that firms were indifferent between sharing all their information and no sharing at all. They consider fulfilled expectations (Bayesian-Nash) equilibrium. Clarke(1983a) showed that the result hinged on the strong equilibrium concept which required that firms ex-ante market expectations were realised ex-post. Clarke(1983a) and the papers which followed used the less restrictive Bayesian-Nash (or rational expectations) equilibrium which only requires that decisions are based on best Bayes estimates of the other firms information. Clarke(1983a) found that firms in general had no incentives to share information. Clarke(1983b) generalised this to a n-firm oligopoly model and showed that there is never a mutual incentive for all firms in the industry to share their information unless they may cooperate on their strategy choice once information has been shared. Vives(1984) generalised the model to the case of symmetric differentiated goods. He showed that if goods are complements, or in the conventional case of competitive goods, if firms were price setters then firms have an incentive to share information. Gal-Or(1985) generalised Clarke(1983b) to the case where signals may be correlated and firms are allowed partial revelation of their information. Still firms have no incentives to share information. Finally Li(1986) generalised the results to allow for all distributions for which the posterior expectation is linear in the recieved signal.

In a Cournot model with random marginal costs and a perfect signal on own costs, Okada(1982) showed that each firm would choose to reveal its private cost information to its rival. Gal-Or(1986) extended this to allow noisy signals and partial revelation and showed that firms have an incentive to share information if goods are substitutes and firms quantity setters, or if goods are complements and firms price setters. Thus Gal-Or reverse the results found in Vives(1984). Shapiro(1986) generalised the model to a n-firm Cournot oligopoly and cases of asymmetry across firms. Shapiro also explicitly modelled the incentives of firms to join a trade association that exchanged cost information. Li(1986) extended Shapiro(1986) by showing that perfect revelation is the unique equilibrium.

Finally Harris and Lewis(1982) consider a two-stage duopoly model where firms first choose investment and then output level. A priori firms are uncertain about the intercept in a common linear demand function. Prior to choosing investment firms observe a positively correlated signal on the random variable. Prior to choosing output, the realisation of the random variable is made known to both. They show the existence and uniqueness of a pure strategy equilibrium. Among other they find that firms may not be better off by observing a more precise signal and that firms prefer the signal to be a bit noisy. Further, if the signals are positively correlated, then more precise signals are preferred, the reverse is true when signals are negatively correlated. This also implies that the incentive to share information diminishes as the correlation between signals decrease and become negative.

The thesis contains five major chapters together with this introduction and a short conclusion. In chapter II we show that the results that there are no incentives for sharing demand information may be reversed if firms are risk-averse because in that case information sharing benefits firms as it reduces the variance of the random variables. Assuming that firms have constant absolute risk aversion, we identify cases where firms prefer to share information. This result does not necessarily require that the firms are very risk averse, i.e. that their utility function is very concave. This indicates that the results in the literature may hinge on the functional forms. Further in a working paper Nalebuff and Zeckhauser(1986) show that the results also hinges on the assumed information structure. All in all this implies that the results should be interpreted with care.

In chapter III we take up the point of Clarke that firms only share information if they can coordinate their strategy choice. To shed some light on this we consider a model with endogeneous cartel formation. We identify cases where private information caused a disincentive to cartel formation. These cases were those where the cartel cannot avoid disseminating most of the shared information to firms not in the cartel. The main implication of this is that information sharing may be detrimental to collusive arrangements and we cannot be sure that information sharing leads to a greater degree of collusion.

In chapter IV and V we consider a two-stage model in which firms first choose capacity, then prices. In this model both voluntary and involuntary information transmission is possible because the strategy choice in the first stage may contain information. Chapter IV contains the analysis of a two stage model under uncertainty. It is shown that no pure strategy equilibrium exists. A mixed strategy exists, and this is worked out for a specific distribution of the random variable.

Chapter V invokes a more sophisticated conjecture on behalf of the firms which ensures that a pure strategy equilibrium exists. In solving this model,we show that if capacity is exogeneously given, an endogeneous Stackelberg leader exist. We consider asymmetric information and show that the best informed do not in general want to share its information with the worse informed. But more surprising there are cases where the worse informed do not want to have the information of the other firm. It seems to be the case that both firms prefer both to be totally ignorant in the first stage because this biases the capacities downwards, and totally informed in the second stage as this ensures an efficient choice of prices.

The transmission of information involuntarily is interesting as firms do use resources on monitoring competitors, and resources are spent on industrial espionage. We therefore attempt to model situations where firms via their plans transmit part or all of their information. In a joint paper with Norman J. Ireland, chapter VI, we show that if firms can learn each others output plans, we can rationalise positive consistent conjectures even in a simple model. This result comes about because a production plan carries two messages. Firstly how much the firm plans to produce and secondly how the firm views the strength of the market demand. Hence a firm may be so heartened by an expansive plans of the other firms that it itself increase its actual output.

In summary, we have shown that private information and the transmission thereof can have very marked effects on the results of even simple duopoly and oligopoly models. A major problem in these models is the difficulty in solving even the most simple cases. Even more troublesome is the fact that changes in functional forms, information sets or stochastic formulations may have quite dramatic effects on the results. This is borne out by even a small degree of risk aversion reversing the results in the literature. This would seem to suggest that the selection of models are (more than ever) an empirical matter. Whereas this is to a large extend true, the models analysed in this thesis point to some severe problems with empirical analysis. This we shall return to in chapter VII which contains the general conclusions.

CHAPTER II

Risk-Averse Duopolists and Voluntary Information Transmission.

1. Introduction.

In the literature it is by now well established that Cournot duopolists producing a homogeneous good do not wish to share information about a random element in demand if they are constrained to acting non-cooperatively, see e.g. Clarke(1983a,b), Gal-Or(1985a), Li(1985) and Vives(1984). Clarke(1983b) concludes:

> "If all industry firms are observed to pool information without paying each other compensation, they must be setting quantities cooperatively on the basis of the homogenized information. Hence information-pooling mechanisms like trade associations can be considered *prima facie* evidence that firms are illegally cooperating to restrict output",(Clarke,1983b, p.392, his italics)

This seems a very strong conclusion to draw from a very simple model. One of the simplifying assumptions of the previous models is that firms are risk-neutral. Hence firms do not gain explicitly from the reduced variance in the conditional expectations of the random variable after information has been shared. By introducing risk aversion the conclusion may be reversed.

we consider a simple model of two firms producing a homogeneous good and facing a demand curve having a random intercept. Firms are assumed to choose their output level to maximise expected utility of profits. For simplicity, we assume that firms have constant absolute risk aversion. Prior to choosing output each firm observe a private signal on the stochastic parameter. This is common knowledge. It is the information contained in this signal which firms may share.

In solving the model, we will be looking for Bayesian-Nash (or rational expectations) equilibria. The problem is that, when firms have private information, firms must form expectations (or conjectures) about how other firms react to their own private information. In the literature it is commonplace to require that expectations are rational in the sense that they are not rejected ex post. In the context of voluntary information sharing, firms cannot evaluate the effects of sharing if they have no expectations as to how the other firms react to the new information. The requirement of rational (or consistent) expectations is naturally extreme, but there seem at present no serious alternative.

By way of solving the model, we also consider the effect of uncertainty and private information on risk-averse duopolists. The setup allows us to consider two special cases. The case where firms observes a perfect signal corresponds to the case where firms are informed about the realisation of the stochastic parameter prior to choosing output. The case where firms observes a useless signal corresponds to the case where firms receives no information on the realisation until after their output choice.

The chapter is organised as follows. In section 2 we set up and solve the model. In section 3 we consider the incentives for firms to share their private information. Two cases are considered. In the first, section 3.1, a firm cannot obtain the information of the other firm unless it submits its own information. The question asked is whether it is in the common interest of the firms to share information. In the second case, section 3.2, we consider unilatteral information sharing, establishing whether it is in the private interest of a firm to share its information with the other firm. An affirmative answer in the second case naturally constitutes a stronger result than in the first case. Note, though, that if firms have a public but not private incentive to share information, there must be incentives to set up an institution or a third party, which can enforce that both firms share their information.

Section 4 concludes.

2. The model.

Consider the following duopoly model with imperfect information. Demand is given by:

$$P = \alpha - Q = \alpha - q_1 - q_2 \quad ; \quad \alpha \sim N(\bar{\alpha}, \sigma_\alpha^2). \tag{1}$$

where q_i is output of firm i. $\bar{\alpha}$ can be interpreted as the firms' prior estimate of α. Costs are assumed constant per unit and subsumed in α. Assume that firms are risk averse and assume that they maximise expected utility of profits given their information. We choose the simplest form of utility function. If firms have constant absolute risk aversion, this corresponds to firms maximising:

$$E[U(\Pi_i)|I_i] = E(\Pi_i|I_i) - \frac{a}{2}\cdot VAR(\Pi_i|I_i) \qquad i=1,2 \tag{2}$$

with respect to q_i, where I_i is the information set of firm i, and a is a positive constant, the Arrow-Pratt measure of the degree of absolute risk aversion, for simplicity and symmetry assumed identical for both firms.

Given (1) we can write (2) as:

$$E[U(\Pi_i)|I_i] = q_i\cdot E(\alpha|I_i) - q_i^2 - q_i\cdot E(q_j|I_i)$$

$$- \frac{a}{2}\cdot q_i^2\cdot [VAR(\alpha|I_i) + VAR(q_j|I_i) - 2\cdot COV(\alpha, q_j|I_i)] \tag{3}$$

First order conditions associated with this are:

$$q_i = \frac{E(\alpha|I_i) - E(q_j|I_i)}{2+a\cdot[VAR(\alpha|I_i)+VAR(q_j|I_i)-2\cdot COV(\alpha,q_j|I_i)]} \qquad i=1,2 \qquad (4)$$

In accordance with the literature on information sharing, assume that firm i observes a private signal on α prior to choosing its output level. The signals of the firms are uncorrelated and are both unbiased estimators of α. They are given by

$$S_i = \alpha + \epsilon_i \qquad i=1,2 \qquad (5)$$

where $\epsilon_i \sim N(0,\sigma_\epsilon^2)$ and $COV(\epsilon_1,\epsilon_2)=0$. These signals could be interpreted as private forecasts. As α and ϵ_i are both normal, the distribution of α conditional on the signal is normal and given by:[1)]

$$\alpha|S_i \sim N\left[t\cdot S_i+(1-t)\cdot\bar{\alpha}\ ,\ (1-t)\cdot\sigma_\alpha^2\right] \qquad (6)$$

where

$$t = \frac{\sigma_\alpha^2}{\sigma_\alpha^2 + \sigma_\epsilon^2} \qquad (7)$$

Note that t is a measure of the precision (or quality) of the private information. The closer is t to 1, the more precise is the signal. The information set of firm i when no information is shared then become $I_i = \{S_i\}$. Given this we can write (4) as

$$q_i = \frac{E(\alpha|S_i) - E(q_j|S_i)}{2+a\cdot[VAR(\alpha|S_i)+VAR(q_j|S_i)-2\cdot COV(\alpha,q_j|S_i)]} \tag{8}$$

As α is assumed normal, variances and covariances are independent of the realisations of the stochastic variables. The actual observation of S_i does not alter the variance of α. The variance is affected by the knowledge that a signal of the form (5) will be observed. Hence, from (8), q_i is a linear function of the conditional expectations of α and q_j. This makes the conjecture that a firm's choice of output is linear in its signal reasonable.[2)] Specifically, assume that firm i expects firm j's output to depend linearly on its information in the following way:

$$q_j^e = c_0\cdot\bar{\alpha} + c_1\cdot(S_j-\bar{\alpha}) \tag{9}$$

where c_0 and c_1 are positive constants whos value is to be determined below. Using (9) we find:

$$E(q_j|S_i)=(c_0 - c_1)\cdot\bar{\alpha} + c_1\cdot E(\alpha|S_i) \tag{10}$$

$$VAR(q_j|S_i)=(1-t)\cdot VAR(q_i)=(1-t)\cdot c_1^2\cdot VAR(S_j)=\frac{1-t}{t}\cdot c_1^2\cdot\sigma_\alpha^2 \tag{11}$$

$$COV(\alpha,q_j|S_i) = (1-t)\cdot COV(\alpha,q_j) = (1-t)\cdot c_1\cdot\sigma_\alpha^2 \tag{12}$$

Using (6), (9)-(12), we can write (4) as

$$q_i = \frac{(1-c_0)\cdot\bar{\alpha} + (1-c_1)\cdot t\cdot(S_i-\bar{\alpha})}{2 + a\cdot\left[1 + \frac{c_1^2}{t} - 2\cdot c_1\right]\cdot(1-t)\cdot\sigma_\alpha^2} \tag{13}$$

In a Bayesian-Nash (rational expectations) equilibrium, the expected response (9) and the actual response (13) must coincide. This implies:

$$c_0 = \frac{1 - c_0}{2 + a\cdot\left[1 + \frac{c_1^2}{t} - 2\cdot c_1\right]\cdot(1-t)\cdot\sigma_\alpha^2} \tag{14}$$

$$c_1 = \frac{(1 - c_1)\cdot t}{2 + a\cdot\left[1 + \frac{c_1^2}{t} - 2\cdot c_1\right]\cdot(1-t)\cdot\sigma_\alpha^2} \tag{15}$$

Lemma 1. For $c_0>0$, $c_1>0$, there exists an unique pair c_0, c_1 solving (14) and (15). Further $c_1 \leq c_0 \leq \frac{1}{3}$.

Proof:

Rewrite (14) and (15) as:

$$c_0 = \frac{c_1}{t + (1-t)\cdot c_1} \tag{16}$$

This implies that $c_0 \geq c_1$ with equality for $t=1$. Further, for any c_1, there exists an unique c_0.

To find c_1, use (15) to define:

$$f(c_1, t, \cdot) \equiv \left[\frac{c_1^3}{t} - 2c_1^2 + c_1\right]\cdot(1-t)\cdot a\cdot\sigma_a^2 + (2+t)\cdot c_1 - t = 0 \tag{17}$$

This is a third-order polynomial having at most three roots. We can put restrictions on c_1. Firstly it must be non-negative.

Otherwise from (9) an increase in expected demand leads to a reduction in output. The first term in (17) is non-negative because $t \leq 1$. Hence the last two terms must be non-positive implying:

$$c_1 \leq \frac{t}{2+t}$$

As c_0 is increasing in c_1 this also implies

$$c_0 \leq \frac{1}{3}$$

which gives us the second part of the lemma, and puts bound on the possible values of c_1. To show the first part, we note that

$$f(\cdot) = t < 0 \qquad \text{for } c_1 = 0$$
$$f(\cdot) > 0 \qquad \text{for } c_1 = \frac{t}{2+t}$$

Thus for some $0 \leq c_1 \leq \frac{t}{2t}$, $f(\ ) = 0$. To show uniqueness, we must show that $f(\ )$ is monotone in c_1 over the permissible range of c_1. From (17):

$$\frac{\partial f}{\partial c_1} = (1-t)\left[\frac{3}{t}\cdot c_1^2 - 4\cdot c_1 + 1\right]\cdot a\cdot\sigma_a^2 + (2+t)$$

For $0 \leq c_1 \leq \frac{t}{2+t}$ we have $\frac{\partial f}{\partial c_1} > 0$.

Hence there exists an unique equilibrium value of c_1 lying between 0 and $\frac{t}{2+t}$. Further, from (16) there exists a unique c_0. This completes the proof.

Thus we have proven the existence of an equilibrium in our model. Further, using (14) and (15) and implicit function theorem, we can show that

$$\frac{\partial c_0}{\partial t} \geq 0 \qquad \frac{dc_1}{dt} \geq 0 \tag{18}$$

Recall that t is a measure of the quality of the private information. Thus (16) implies that equilibrium firm output as given by (9) is increasing in the precision of the private information. Conversely, one can show that

$$\frac{\partial c_0}{\partial a} \leq 0 \qquad \frac{\partial c_1}{\partial a} \leq 0 \tag{19}$$

$$\frac{\partial c_0}{\partial \sigma_\alpha^2} \leq 0 \qquad \frac{\partial c_1}{\partial \sigma_\alpha^2} \leq 0 \tag{20}$$

Thus the more risk-averse, or the higher the variance, the lower is equilibrium output.

The result in (16) suggests that profits may be decreasing in t because we have two counterbalancing effects. Better information increases output which depresses profits, but it also decrease the variance of profits which increases the expected utility of profits. The effect of a change in the degree of risk-aversion would tend to be opposite of the effects from a change in t because an increase in a leads to a fall in output and hence an increase in expected profits. On the other hand, firms become increasingly worried about the variance of profits. The effects from a change in σ_α^2 are harder to assess as these also leads to changes in t which again affects c_0 and c_1. Hence we can not get clear comparative static results on the effect on expected utility of profits from changes in t, a and σ_α^2.

Although we cannot get explicit expressions for c_0 and c_1, we can, use (9) to write expected utility of profits as:

$$E(U(\Pi_i)) = c_0 \cdot (1-2c_0) \cdot \bar{\alpha}^2 + c_1 \cdot \left[1 - \frac{c_1}{t} - c_1\right] \cdot \sigma_\alpha^2$$

$$- \frac{a\sigma_\alpha^2}{2} \cdot \left[\left[\frac{c_1}{t} \cdot \left[5 \cdot c_0^2 \cdot c_1 + 4 \cdot c_0^2 - 2 \cdot c_0 \cdot c_1 + c_1 - 2 \cdot c_0\right] + c_0^2 - 2 \cdot c_0^2 \cdot c_1\right] \cdot \bar{\alpha}^2\right.$$

$$\left. + \left[\frac{c_1^4}{t^2} + \frac{c_1^2}{t} \cdot \left[c_1^2 - 6 \cdot c_1 + 1\right] + 2 \cdot c_1^3 \cdot \left[c_1 + 1\right]\right] \cdot \sigma_\alpha^2\right] \quad (21)$$

Before turning to information transmission, we consider three special cases of (21): t=0, t=1 and a=0.

If the information received is useless, i.e. $\sigma_\varepsilon \to \infty$ and t=0, then from (14) and (15), $c_0 = \dfrac{1}{3 + a \cdot \sigma_\alpha^2}$ and $c_1 = 0$. The expected utility of profits is:

$$E(U(\Pi_i)) = \frac{1 + \frac{1}{2} \cdot a \cdot \sigma_\alpha^2}{\left[3 + a \cdot \sigma_\alpha^2\right]^2} \cdot \bar{\alpha}^2 \qquad (22)$$

If the information received is perfect, then t=1 and $c_0 = c_1 = \frac{1}{3}$ and we get:

$$E(U(\Pi_i)) = \frac{1}{9}\cdot\left[1 - \frac{1}{9}\cdot a\cdot\sigma_\alpha^2\right]\cdot\bar{\alpha}^2 + \frac{1}{9}\cdot\left[1 - \frac{1}{18}\cdot a\cdot\sigma_\alpha^2\right]\cdot\sigma_\alpha^2 \qquad (23)$$

(22) and (23) are interesting when one recalls the debate in the literature on price variation in a competitive model. An early result by Oi(1961) showed that firms preferred greater variance in price if the realisation of the price was known prior to their output choice. This corresponds to the first case above, equation (22). Later Sandmo(1971) and Ishii(1977) showed that if the realisation was known only after output was choosen, corresponding to equation (23), firms preferred less variation in the price. Here we get less clear results. From (20), we see that if a is large ($a \geq 2$ sufficient) we get the same result as Oi(1961). The results on (23) are ambiguous. Thus with few firms acting strategically, we cannot reproduce the results from the competitive literature.

The final special case, which will be used in section 3 as a bench-mark, is risk neutrality, $a=0$. From (14) and (15) we get $c_0 = \frac{1}{3}$ and $c_1 = \frac{t}{2+t}$. Profits become:

$$E(\Pi_i) = \frac{1}{9}\cdot\bar{\alpha}^2 + \frac{t}{(2+t)^2}\cdot\sigma_\alpha^2 \qquad (24)$$

This is the case considered in the literature sofar.

3. Information sharing.

We consider two sharing systems. In both cases we assume that the information is transmitted via a third party, an auditor who can determine whether or not the information is true. This ensures truth-telling and allows us to disregard problems of strategic misinformation. As we consider only one-shot (or atemporal) games, the model is not suitable for analysing cases where firms deliberately misinform. It is easy to see that in a one-shot game where truth-telling is not required, both will claim that demand is low. When demand is actually low, both want to avoid over-production. When demand is high, both want the other to leave it a larger share of the market.

It is worth mentioning that such third parties actually exists. Trade associations can collect data from firms and, to the extend that the realisation can be observed ex post, punish any firm found lying (e.g. by banishing the firm from the association). Another institution is statistical offices. These offices sometimes relies on firms reports in order to produce their statistics faster than could be done by using information from e.g. the excise office. Firms are sometimes given an incentive to report their date by being given a sneak previev of the data (compare with the second case below). Given that the problem is verification, it is further concievable that an actual independent auditor is used. The basic point is that if firms have an incentiv to share if they do so truthfully, then if costless or cheap mechanisms exists that ensure this, these will be set up.

In the first case analysed, firms only get access to the information of the other firm if it itself submits its information. Hence the two situations to compare is "no sharing" with "both sharing". In the second case, a firm get the information recieved by the auditor regardless of whether it has submitted any information. In this case we consider the model as a two stage game, where firms in the first stage decides whether or not to submit its information to the auditor. Under risk neutrality, no information transmission takes place in either case. If firms are risk averse we shall see below that this is no longer generally true.

3.1 Both share information.

If firms share their information, their information set become

$$I_i = \{S_1, S_2\} \qquad i=1,2 \tag{25}$$

We can construct a composite signal S summarising the information of the two signals as follows:

$$S = \alpha + \frac{1}{2}\cdot\epsilon_1 + \frac{1}{2}\cdot\epsilon_2 \tag{26}$$

$$\mathrm{VAR}(S) = \sigma_\alpha^2 + \frac{1}{2}\cdot\sigma_\epsilon^2 \tag{27}$$

The general formulas are given in appendix II.A. When both share the information, t is redefined as

$$\hat{t} = \frac{2 \cdot t}{1+t} > t \tag{28}$$

As above we can calculate the expected utility of profits. The difference from (23) is not only that t is redefined, but more importantly that the output of the two firms are closer correlated.

$$E(U(\hat{\Pi}_i)) = \hat{c}_0 \cdot (1-2 \cdot \hat{c}_0) \cdot \bar{\alpha}^2 + \hat{c}_1 \cdot \left[1 - \frac{2}{\hat{t}} \cdot \hat{c}_1\right] \cdot \sigma_\alpha^2$$

$$- \frac{a \cdot \sigma_\alpha^2}{2} \cdot \left[\left[\hat{c}_0^2 + 2 \cdot \hat{c}_0 \cdot \hat{c}_1 \cdot (1-4 \cdot \hat{c}_0) + \hat{c}_1^2 \cdot (1-4 \cdot \hat{c}_0) \cdot \frac{1}{\hat{t}}\right] \cdot \bar{\alpha}^2 \right.$$

$$\left. + \left[4 \cdot \frac{\hat{c}_1^4}{\hat{t}^2} - 4 \cdot \hat{c}_1^3 \cdot \frac{2-\hat{t}}{\hat{t}} + \frac{\hat{c}_1^2}{\hat{t}}\right] \cdot \sigma_\alpha^2 \right] \tag{29}$$

where $\hat{c}_0$ and $\hat{c}_1$ are evaluated at equilibrium.

Let us first look at the special case of risk-neutrality, a=0:

$$\hat{c}_0 = 1/3 \qquad \hat{c}_1 = \frac{\hat{t}}{2+\hat{t}}$$

$$E(U(\hat{\Pi}_i)) = \frac{1}{9} \cdot \bar{\alpha}^2 + \frac{\hat{t}^2}{(2+\hat{t})^2} \cdot \sigma_\alpha^2 \tag{30}$$

Using (28) we find comparing (24) and (30):

$$E(U(\hat{\Pi}_i)) = \frac{1}{9} \cdot \bar{\alpha}^2 + \left[\frac{t}{1+2t}\right] \cdot \sigma_\alpha^2 < E(U(\Pi_i)) \tag{31}$$

So even if they both share their information, information sharing is sub-optimal under risk-neutrality.

For $a > 0$, we no longer have an unambiguous result, so we turn to some numerical examples. We are looking for combinations of t, $\bar{\alpha}$, a and σ_{α}^{2} such that information sharing is preferred by the two firms.

Generally we would like to have risk sharing preferred for as small an a as possible because the smaller is a the closer we are to the case of risk neutrality. Interest then centres on $\bar{\alpha}$ and σ_{α}^{2}. Given the assumed normal distribution, we can place some restrictions on these, loosely speaking the latter must be small relative to the former to avoid negative realisations of demand. In order to make the probability of observing a negative realisation of the demand intercept small, we choose $\bar{\alpha}$ large relative to σ_{α}^{2}. Now:

$$\text{Prob}\{\alpha<0\} < .05 \quad \Rightarrow \quad \frac{\bar{\alpha}^{2}}{\sigma_{\alpha}^{2}} > 2.27$$

$$\text{Prob}\{\alpha<0\} < .01 \quad \Rightarrow \quad \frac{\bar{\alpha}^{2}}{\sigma_{\alpha}^{2}} > 5.43$$

$$\text{Prob}\{\alpha<0\} < .001 \quad \Rightarrow \quad \frac{\bar{\alpha}^{2}}{\sigma_{\alpha}^{2}} > 9.55$$

For expositional purposes, choose $a = A/\sigma_{\alpha}^{2}$, where A is some positive constant. This implies that $a\cdot\sigma_{\alpha}^{2}$ becomes constant. Then, given A $(=a\cdot\sigma_{\alpha}^{2})$ and t, we can find c_0 and c_1 and express expected utility of profits as a linear function of $\bar{\alpha}^{2}$ and σ_{α}^{2}. Also, letting P and S denotes private information and sharing respectively, we can write expected utility of profits given by either (21) or (29) as:

$$E(U(\Pi_i^P)) = \xi_P \cdot \bar{\alpha}^2 + \zeta_P \cdot \sigma_\alpha^2 \tag{32}$$

$$E(U(\Pi_i^S)) = \xi_S \cdot \bar{\alpha}^2 + \zeta_S \cdot \sigma_\alpha^2 \tag{33}$$

where ξ_i and ζ_i, i=s,p are constants. Using equation (32) and (33) we can find the combination of $\bar{\alpha}^2$ and σ_α^2 for which firm i is indifferent between sharing and not sharing, $\bar{\alpha}^2 = \Lambda \cdot \sigma_\alpha^2$, where Λ is given by $\Lambda = (\zeta_p - \zeta_s)/(\xi_s - \xi_p)$. Thus any $\bar{\alpha}^2/\sigma_\alpha^2 < \Lambda$ implies that firm i have an incentive to share its information with firm j.

Below in table 1 we have worked through some numerical examples, with Λ given if relevant.

<u>Table 1. Profits of firm i when both share information.</u>

$a\sigma_\alpha^2$	t	case	ξ	ζ	Λ
0.00	0.5	P	1/9	.08	
		S	1/9	.0625	-
0.01	0.1	P	.11091	.02593	
		S	.11089	.00707	
	0.5	P	.11086	.07994	
		S	.11085	.06257	
	0.9	P	.11086	.10698	
		S	.11086	.10328	1650.00
0.10	0.1	P	.10903	.02187	
		S	.10891	.00808	
	0.5	P	.10856	.07940	
		S	.10853	.06320	
	0.9	P	.10858	.10661	
		S	.10861	.10296	155.00
1.00	0.1	P	.09081	.01639	
		S	.08963	.01282	
	0.5	P	.08391	.07272	
		S	.08413	.06708	25.82
	0.9	P	.08519	.10269	
		S	.08576	.09960	5.42
0.01	1.0	P	.11086	.11105	
		S	.11086	.11105	-

From table 1 note that if t=0.9, a=0.01, $\sigma_{\alpha}^{2}=1$ and $\bar{\alpha} > 41$ information sharing is preferred. Thus it is possible to reverse the result obtained under risk-neutrality even with a low absolute degree of risk aversion provided that $\bar{\alpha}$ is sufficiently greater than σ_{α}^{2}. Note further that the higher is a, the more readily do we get information sharing.

To confirm the idea that for t and a large, the critical value Δ is small and information transmission likely, we have shown Δ as a function of t in figur 1.

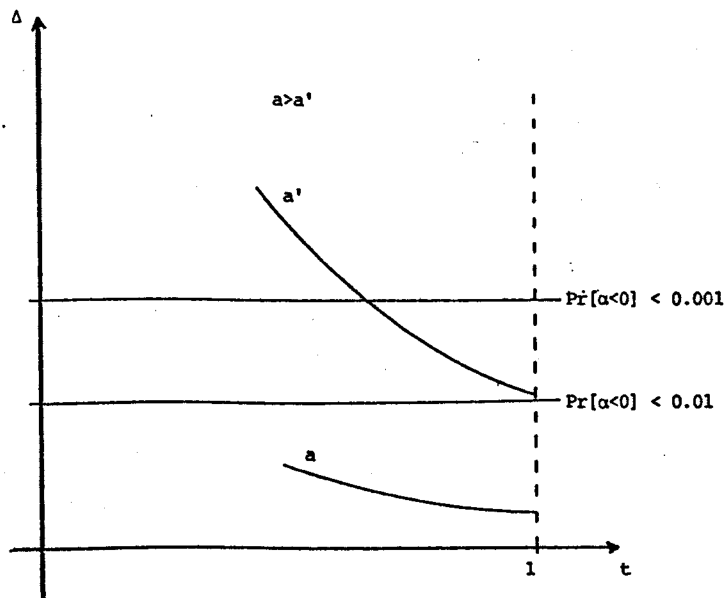

Figure 1.

Thus in the case where firms are very risk-averse, they are willing to share information, supporting the initial intuition. Further if the private information is fairly precise firms have an incentive to share information. This latter point should be interpreted with some care, as much of the value from sharing when t is high comes from the assumption that the private information is uncorrelated.

3.2 Unilateral sharing.

In this case a firm recieve the pooled information from the auditor indepenent of whether or not it has itself transmitted any information. We then have to consider the incentives to unilaterally transmit their information. Assume that only firm i transmit its information. The information sets then become:

$$I_i = \{S_i\} \qquad I_j = \{S_i, S_j\}$$

The problem becomes less tractable because we have to look at asymmetric solutions to (14) and (15). Given that a solution exists we can write down expected utility of profits of the firm transmitting its information unilaterally as:

$$E(U(\tilde{\Pi}_i)) = c_{0i}\cdot(1-c_{0i}-c_{0j})\cdot\bar{\alpha}^2 + c_{1i}\cdot\left[1 - \frac{1}{t_i}\cdot c_{1i} - c_{1j}\right]\cdot\sigma_\alpha^2$$

$$- \frac{1}{2}\cdot a\cdot\sigma_\alpha^2\cdot\left[\left[c_{0i}^2\cdot c_{1j}^2\cdot\frac{1}{t_j} + (1-2\cdot c_{0i}-c_{0j})^2\cdot c_{1i}^2\cdot\frac{1}{t_i} + c_{0i}^2\right.\right.$$

$$\left. - 2\cdot c_{0i}^2\cdot c_{1j} - 2\cdot c_{0i}\cdot c_{1i}\cdot(1+c_{1j})\cdot(1-2\cdot c_{0i}-c_{0j})\right]\cdot\bar{\alpha}^2$$

$$+ \left[c_{1i}^4\cdot\frac{1}{t_i^2} - c_{1i}^3\cdot c_{1j}\cdot\frac{2}{t_i} + c_{1i}^2\cdot c_{1j}^2\cdot\left[\frac{3}{t_i\cdot t_j} - 2\right]\right.$$

$$\left.\left. - c_{1i}^3\cdot 2\cdot\frac{2-t_i}{t_i} - c_{1i}^2\cdot c_{1j}\cdot\frac{2}{t_i} + c_{1i}^2\cdot\frac{1}{t_i}\right]\cdot\sigma_\alpha^2\right] \qquad (34)$$

where c_{0i}, c_{0j}, c_{1i} and c_{1j} are evaluated at equilibrium.

Let us first consider risk-neutrality.

$$c_{0i}=c_{0j}=1/3 \qquad c_{1i}=\frac{2t_i-t_it_j}{4-t_it_j} \qquad c_{1j}=\frac{2t_j-t_it_j}{4-t_it_j}$$

$$E(U(\tilde{\Pi}_i)) = \frac{1}{9}\cdot\bar{\alpha}^2 + \frac{t_i\cdot(2/t_j)^2}{(4-t_it_j)^2}\cdot\sigma_\alpha^2 \tag{35}$$

Now $t_i=t$ and $t_j=2t/(1+t)$. Comparing (35) with (24) we see that sharing is not profitable for firm i, hence in combination with (29) we have the well known result that under risk-neutrality, is a dominating strategy not to share information.

Turning to a > 0, we work through the same numerical examples as presented in table 1. The results are presented in table 2 below:

Table 2. Profits of the transmitting firm from unilatteral information sharing.

$a\sigma_{\alpha}^{2}$	t	case	ξ	ζ	Λ
0.00	0.5	P	1/9	.08	
		S	1/9	.0625	-
0.01	0.1	P	.11091	.02593	
		S	.11090	.02078	
	0.5	P	.11086	.07994	
		S	.11087	.06607	2021.96
	0.9	P	.11086	.10698	
		S	.11086	.10 61	2301.79
0.10	0.1	P	.10903	.02187	
		S	.10909	.02019	29.04
	0.5	P	.10856	.07940	
		S	.10876	.06563	72.20
	0.9	P	.10858	.10661	
		S	.10865	.10019	99.66
1.00	0.1	P	.09081	.01639	
		S	.09456	.01555	0.22
	0.5	P	.08391	.07272	
		S	.09474	.06070	1.11
	0.9	P	.08519	.10269	
		S	.08901	.09463	2.12
0.01	1.0	P	.11086	.11105	
		S	.11086	.11105	-

We see that even if a is very small, we can find combinations of $\bar{\alpha}$ and σ_{α}^{2} such that unilatteral information sharing is preferred by the transmitting firm. Further the critical value of Λ to the transmitting firm seems to be increasing in a for fixed σ_{α}^{2} and t, and decreasing in t. Note that the picture here is reversed in the case of varying t. This is naturally caused by the fact that if you are the only firm transmitting, then the concequence is small when your information is imprecise. You are not giving so much away.

We then consider the incentives for firm j to submit its information once it has recieved the information of firm i. The numerical examples are presented in table 3 below, where the case 1S denotes only firm i sharing and 2S denotes both sharing.

Table 3. Profits of the recieving firm.

$a\sigma_\alpha^2$	t	case	ξ	ζ	Δ
0.00	0.5	1S	1/9	.08	
		2S	1/9	.06612	-
0.01	0.1	1S	.110893	.04127	
		2S	.110892	.00707	
	0.5	1S	.110845	.11153	
		2S	.110855	.06257	5264.34
	0.9	1S	.110858	.11569	
		2S	.110862	.10328	3877.25
0.10	0.1	1S	.10902	.04013	
		2S	.10891	.00808	
	0.5	1S	.10853	.11115	
		2S	.10853	.06320	
	0.9	1S	.10859	.11540	
		2S	.10861	.10296	615.83
1.00	0.1	1S	.09539	.03084	
		2S	.08963	.01282	
	0.5	1S	.09245	.10519	
		2S	.08413	.06704	
	0.9	1S	.08669	.11247	
		2S	.08576	.09960	
0.01	1.0	1S	.11086	.11105	
		2S	.11086	.11105	-

Here we see that in all but a few cases (e.g. a=0.1, t=0.9, σ_α^2=1 and $\bar{\alpha}$=26) firm j would not prefer to submit its information once it has got the information of firm i. Thus transmission is only in some, possibly few, cases a dominant strategy if a firm can get the information of the other firm without having to submit its own information (but such cases do exist, e.g. t=.9 and A=.01). This implies a potential need for a coordination devise because of the cases where both firms would benefit from sharing if only they could commit to both submitting the information. The coordination could be supplied by the auditor by simply requiring that a firm to obtain some information must supply its own.

4. Concluding remarks

Two things have been shown in this paper. Firstly, we found that the output level of risk averse Cournot duopolists was increasing in the quality of their private information and decreasing in their perception of risk. The effect on profits was less clear, but from the numerical results in table 1 - 3, it seems that expected utility of profits are decreasing in the degree of absolute risk aversion.

Secondly, we have shown that once one allows for risk-aversion, it is no longer generally true that duopolists do not want to share information. This was shown using numerical examples, and is yet another example of the non-robustness of results derived in models analysing the effect of private information.

We also found cases where firms had an incentive to share their information, but where this would require that they could only obtain the shared information if they submitted their own. As argued above, this need not be restrictive and can tentatively be used to explain some institutional arrangements, as well as the practise of some institutions..

Besides having to resort to numerical examples, the paper makes a number of heroic assumptions regarding both the specification of the utility function and the distributions of the stochastic variables. Firstly, the aim was to provide examples where the results in the litterature did not hold. To this end, examples are sufficient. Secondly, we did not need the utility functions to be very concave to get the result, so we would expect the results to hold for other functions as well. Thirdly, we could have used other distributions. The important thing is the linear update rule (6). As shown in Ericson(1969), see also Li(1986), this rule also holds with other distributions which do not suffer from the problem of having positive probabilities of negative realisations.

Finally, the model analysed have been an atemporal Cournot oligopoly model. we should like to point out a problem with these models when private information is introduced. If we consider a dynamic model with learning as e.g. Kirman(1983), where firms in a duopoly over time learn about their residual demand, then although firms' estimates of the parameters may converge, strong assumptions are needed to insure that they converge to the actual parameters of the residual demand. Thus even in a rational expectations model, the outcome may differ from the Cournot outcome because the estimates are biased. This implies that the assumption of unbiased estimates of the random parameter (i.e. (5)) is far stronger than commonly thought, as in ensures that the expected output will be the Cournot output. It would therefore be interesting to consider models where firms have to learn the parameters from past observations on price and their own output level, a task left for future research.

Footnotes:

1) The linearity of the conditional expectation $E(\alpha|S_i)$ does not hinge specifically on the normality assumption. It would be equally true if f.i. the prior was gamma and the signal poisson, or if the prior was beta and the signal binomial, see Ericson(1969).

2) Any non-linearity would arise because firms thought that someone thought that some-one reacted non-linearily. Thus one could concievably get self-fulfilling equilibria which are not affine, but it would require an unconvincing story.

Appendix II.A

In general if S_1, S_2 are bivariat normal with variance-covariance matrix

$$E(S_1, S_2) = \begin{bmatrix} \sigma_1^2 & \sigma_{12} \\ \sigma_{12} & \sigma_2^2 \end{bmatrix}$$

then we can construct a composite signal S summarising the information of the two signals as follows:

$$S = \delta \cdot S_1 + (1 - \delta) \cdot S_2$$

$$VAR(S | S_1, S_2) = \frac{\sigma_1^2 \cdot \sigma_2^2 - (\sigma_{12})^2}{\sigma_1^2 + \sigma_2^2 - 2 \cdot \sigma_{12}}$$

where

$$\delta = \frac{\sigma_2^2 - \sigma_{12}}{\sigma_1^2 + \sigma_2^2 - 2 \cdot \sigma_{12}}$$

CHAPTER III

Stackelberg Leading Cartel and Differential Information.

1. Introduction.

The question which concerns us is to what extend firms have an incentive to share private information. A number of papers, Clarke(1983a,b), Gal-Or(1985a,1986), Novshek and Sonnenshein(1982), Li(1985), Sakai(1985), Shapiro(1985) and Vives(1984) have considered Cournot or Bertrand models where firms were allowed to pool information. The general conclusion to emerge is that in equilibrium firms have no incentives to share information. In these models, firms are not allowed to collude over strategies, only information. Clarke(1983b) points out that if firms are allowed to collude over strategies the result would be reversed. Our aim is to set up a simple model in which firms choose whether or not to share information <u>and</u> collude over the choice of strategies.

To this end, we consider a model of endogenous cartel formation, where the cartel is assumed to pool the information of the members. We use the Stackelberg solution concept with the cartel acting as the Stackelberg leader. The non-cartel members act as Cournot followers, taking the output chosen by the cartel and the other followers as given.[1)] In the case considered here, this solution concept is equivalent to the one suggested in Selten(1973). To obtain a benchmark with which to evaluate the impact of imperfect information and information sharing the model is solved under certainty. This is compared to the solutions under different information structures to see whether different information structures ceteris paribus gives rise to larger or smaller cartels, i.e. whether there is a gain from sharing information.

We consider a model of differential information. No firm believes that any other individual firm has access to better information. Each firm observes a private signal on a random variable, essential for their choice of strategy. In accordance with the literature, this is assumed to be the intercept of a common inverse demand function. A cartel bases its choice of output on its signal, and the expected response of the followers. A follower base its choice of output on its private signal, the other followers' expected response to their signal, and the output of the cartel, which given the sequential nature of the game is known to them with certainty. In general we assume the following timing of events. Prior to observing their signal, firms choose whether or not to belong to a cartel acting as specified above. When the cartel is formed, firms recieve their signal and the cartel chooses its strategy. At this stage it is assumed that no firm will be allowed to leave or join the cartel. Following the disclosure of the strategy of the cartel, the followers choose their strategy non-cooperatively and without communication.

We assume that the n firms in the industry are identical in all important respects, having access to the same technology, producing a homogeneous good, and observing a signal with a common mean and variance. The question is firstly whether a stable cartel exists in such a model. Secondly, what is the maximum number of firms n^* for which it is true that all firms are in the cartel? We shall call this the critical number of firms. This number is of interest because all firms are identical. If only a subset of all firms are in the cartel, the firms outside have higher expected profits than does the cartel members. Although such a cartel can be stable, no process apart from side payment will ensure the establishment of the cartel, because otherwise identical firms would be treated differently. Hence a stable cartel would only emerge for $n \leq n^*$. For the purpose of assessing the effect of information sharing within the cartel, the interest centres around the effect on n^*. If differential information leads to a higher n^*, we will say that information sharing contributes to collusion. If on the other hand it leads to a lower n^*, then information sharing hampers collusion.

In section 2 the model is solved under certainty. It is shown that a unique stable cartel exists and can be characterised fully. As noted above, the Stackelberg solution concept is in this case formally identical to the solution concept proposed in Selten(1973). This solution concept is described in section 2, where the equivalence is further elaborated.

In section 3, a model with differential information in which each firm is assumed to observe a private signal on a random variable is solved. It is assumed that firms act as if they only had access to their own information, i.e. there is no learning.

In section 4 the fact that the strategy of the cartel found in section 3 reveals all its information to the followers is used. This implies that the followers become the better informed, and not surpricingly this gives a disencentive to cartel formation.

In section 5 we sketch a model in which it is not possible for the followers to learn all the information of the leader. We establish necessary conditions for how little of the cartel information the followers are allowed to learn in order that information transmission on its own lead to a larger cartel (i.e. a larger cartel).

Section 6 considers some related papers, Sakai(1984) on Stackelberg games, Clarke(1982) and Shapiro(1985) on Cournot games with collusion.

Section 7 concludes the paper and points at directions for further research.

2. Certainty.

Assume that the industry contains n firms, initially in a Cournot-Nash equilibrium. Entry is not allowed, thus n is fixed. Let demand be linear with the slope normalised to -1.

$$P = \alpha - Q \qquad Q = \sum_{i=1}^{n} q_i \tag{1}$$

where P is the price and q_i output level of firm i. Let marginal costs be constant, for simplicity set equal to zero, implying that price is interpreted as net of marginal costs.

We assume that firms choose whether or not to belong to a cartel prior to choosing their output level. The cartel maximises joint profits and firms get an equal share of this profit. The cartel choose its strategy (output level) proir to the firms outside the cartel. This can be described as a three-stage game version of d'Aspremont et al(1983). In stage 1, firms simultaneously choose whether or not to belong to the cartel. The number of cartel members is made known to all. In stage 2 the cartel choose its output level. This is made known to the other firms. In stage 3 the followers, that is firms not in the cartel, choose their output level simultaneously. The last two stages are simply the Stackelberg leader-follower oligopoly model where the followers are individual Cournot firms.

Looking for a subgame perfect equilibrium we first solve the Stackelberg sub-game for a given number of cartel members k.

If k=0 we get the n-firm Cournot model, the solution to which is:

$$q(0) = \frac{\alpha}{n+1}$$

$$P(0) = \frac{\alpha}{n+1}$$

$$\Pi(0) = \left[\frac{\alpha}{n+1}\right]^2 \tag{2}$$

Let superscript f and c denote follower and cartel firm respectively and let $\Pi^i(k)$, i=f,c denote firm profit of a follower and cartel member respectively if the cartel has k members. The resulting equilibrium depends on k, the number of cartel members.

$$q^c(k) = \frac{\alpha}{2\cdot k} \qquad 1 \le k \le n$$

$$q^f(k) = \frac{\alpha}{2\cdot(n-k+1)} \qquad 1 \le k \le n-1$$

$$P(k) = \frac{\alpha}{2\cdot(n-k+1)} \qquad 1 \le k \le n$$

$$\Pi^c(k) = \frac{\alpha^2}{4\cdot k\cdot(n-k+1)} \qquad 1 \le k \le n \tag{3}$$

$$\Pi^f(k) = \frac{\alpha^2}{4\cdot(n-k+1)^2} \qquad 1 \le k \le n-1 \tag{4}$$

Note that the total cartel output Q^c is always equal to $\alpha/2$, the monopoly output level, independent of k and n. This is only true for this special case where marginal costs are identical. It arises because the resudial demand of the cartel, given the optimal reaction of the fringe (given by $P=\frac{1}{n-k+1}\cdot(\alpha-Q^c)$) is an anti-clockwise rotation of the demand curve around $P(Q)=0$. Hence the first order condition of the cartel is independent of n and k.

An equilibrium in the first stage is a k^* such that firm i is indifferent between changing its position. Thus if firm i belong to the cartel:

$$\Pi^c(k) \geq \Pi^f(k-1) \tag{5}$$

If firm i is a follower:

$$\Pi^f(k) \geq \Pi^c(k+1) \tag{6}$$

Then k^* is an equilibrium if (5) and (6) holds for all i.

By inspection $\frac{\partial \Pi^f}{\partial k} > 0$, $\frac{\partial \Pi^f}{\partial k} > \frac{\partial \Pi^c}{\partial k}$, $\frac{\partial \Pi^c}{\partial k} \gtrless 0$ for $k \gtrless \frac{n+1}{2}$, where we for simplicity treat $\Pi^i(k)$ $(i=c,f)$ as continuous functions of k. Comparing (2) and (3) we see that $\Pi^c(1) > \Pi(0)$. Further $\Pi^f(k) \gtrless \Pi^c(k)$ for $k \gtrless \frac{n+1}{2}$. Finally:

$$\lim_{k\to n} \Pi^f(k) = \frac{\alpha^2}{4} > \Pi^c(n) = \frac{\alpha^2}{4n}$$

From this follows that a k^* exists. For $k < \frac{n+1}{2}$, (6) is violated. At $k = \frac{n+1}{2}$ we have $\Pi^f(k) = \Pi^c(k)$ and (5) holds. If (6) holds, we have an equilibrium. Otherwise we know that (5) must hold for k+1 and we can preceed in an iterative manner until (6) holds or k=n is reached.

In the present case we can find an analytical expression for k^*. Using (3) and (4) we can write (5) and (6) as:

$$k \leq \frac{3 \cdot n+5-\sqrt{n^2-2 \cdot n-7}}{4} \text{ or } k \geq \frac{3 \cdot n+5+\sqrt{n^2-2 \cdot n-7}}{4} \tag{5'}$$

$$\frac{3 \cdot n+1+\sqrt{n^2-2 \cdot n-7}}{4} \geq k \geq \frac{3 \cdot n+1-\sqrt{n^2-2 \cdot n-7}}{4} \tag{6'}$$

For n>4 a stable cartel k is given by

$$\frac{3 \cdot n+5-\sqrt{n^2-2 \cdot n-7}}{4} \geq k \geq \frac{3 \cdot n+1-\sqrt{n^2-2 \cdot n-7}}{4} \tag{7}$$

The distance between the upper and lower bound is equal to one. A stable cartel exists and is unique, because as is shown in appendix III.A, the lower bound in (7) cannot be an integer. The equilibrium can be characterised as follows:[2)]

$$k^* = \begin{cases} n & \text{if } n \leq 4 \\ \left\langle \frac{n+3}{2} \right\rangle & \text{if } n > 4 \end{cases} \tag{8}$$

$$\Pi^c(k^*) = \begin{cases} \frac{1}{n}\cdot\left[\frac{\alpha}{2}\right]^2 & \text{if} \quad n \leq 4 \\ \left[\frac{\alpha}{2}\right]^2 \cdot \frac{1}{\left[n+1-\langle\frac{n+3}{2}\rangle\right]\langle\frac{n+3}{2}\rangle} & \text{if} \quad n > 4 \end{cases} \tag{9}$$

$$\Pi^f(k^*) = \begin{cases} — & \text{if} \quad n \leq 4 \\ \left[\frac{\alpha}{2}\right]^2 \cdot \frac{1}{\left[n+1-\langle\frac{n+3}{2}\rangle\right]^2} & \text{if} \quad n > 4 \end{cases} \tag{10}$$

It is worth noting that we are able to find an explicit analytical solution to the model analytically. This is rarely the case. In the model assuming competitive followers, e.g. d'Aspremont et al(1983) and Donsimoni(1985) this is not possible. This gives us a benchmark with which to compare the case with differential information.

The second thing to note is that n=4 is the highest number of firms for which it is true that all firms will be in the cartel. For n>4 we know from above that $\Pi^f(k^*) > \Pi^c(k^*)$. As firms are initially identical, it is not obvious how such an equilibrium could be reached. This makes $n^*=4$ the interesting benchmark.

2.1 An equivalent solution concept.

Selten(1973) considers the following three stage game model of the endogenous formation of a cartel. In stage 1, firms decide whether or not to participate in cartel negotiations possibly leading to the formation of a cartel. The decision is taken simultaneously and the outcome is made known to all the firms. It is assumed that firms can enter binding quota agreements setting an upper bound to their output level. In stage 2, firms having chosen to participate in the negotiations simultaneously submit a proposed set of quotas for each af the participants. If a set of firms agree on a set of quotas, these firms will constitute a cartel. The outcome of this stage is made known to all firms prior to stage 3, in which firms play a Cournot game subject to the constraints imposed by the quotas.

For the quotas to make sense they must be exactly binding in equilibria. Thus it is as if the cartel has the first mover advantage of a Stackelberg leader, and the cartel will choose a set of quotas identical to the choice of cartel output level chosen by a Stackelberg leading cartel. Thus stage 2 and 3 are equivalent to the two stages in the Stackelberg leader-follower game.

Selten use the concept of perfect equilibrium in this paper implying that the equilibrium must be subgame perfect. As the model is then solved backwards the choice implied in stage 1 corresponds to requirement (5) and (6) above.

The model used in section 2 above is simpler to apply than the Selten(1973) model which gets very complicated once one allows for more general cost or demand structures. The equivalence supports the notion that the Stackelberg solution concept is a possible approximation to models with endogenous cartel formation. The most problematic assumption in Selten(1973) is the assumption of binding quota agreements, corresponding to the assumption in the Stackelberg model that the leader has exogeneously been allotted the first mover advantage. What the Selten(1973) model helps to bring out clearly is the possibility that cartel negotiations can lead to precommitment of the members.

3. Cartel equilibrium under differential information.

Assume that the demand intercept α is random. The inverse demand function is given by:

$$P = \alpha - \sum_{i=1}^{n} q_i \qquad \alpha \sim N(\overline{\alpha}, \sigma_\alpha^2) \tag{11}$$

Before making their strategy decision, each firm observe a private signal S_i, given by:

$$S_i = \alpha + \epsilon_i \tag{12}$$

where $\epsilon_i \sim N(0, \sigma_\epsilon^2)$, $E(\epsilon_i, \epsilon_j)=0$ for $i \neq j$, and $E(\alpha, \epsilon_i)=0$ for all i

As both S_i and α are assumed normal, the expected value of α given S_i is given by:[3)]

$$E(\alpha | S_i) = t \cdot S_i + (1-t) \cdot \overline{\alpha} \tag{13}$$

where

$$t = \frac{\sigma_\alpha^2}{VAR(S_i)} \tag{14}$$

As in section 2 we first solve the n-firm oligopoly model. Firms are assumed risk neutral, maximising expected profits given their information. The problem of a typical firm is:

$$\max_{q_i} E\left[(\alpha - \sum_{j\neq i} q_j - q_i)\cdot q_i | S_i\right]$$

$$= \max_{q_i} \left[q_i\cdot E(\alpha|S_i) - q_i\cdot\sum_{j\neq i} E(q_j|S_i) - q_i^2\right] \tag{15}$$

To solve this a firm must make some conjecture of how other firms react to their signal. The linearity of (11) and (13) makes the guess of a linear response function reasonable

$$q_j^g = h_{0j}\cdot\bar{\alpha} + h_{1j}\cdot(S_j - \bar{\alpha}) \tag{16}$$

As the firms are assumed identical, the coefficients h_{0i} and h_{1i} are identical for all i. Hence

$$\sum_{j\neq i} E(q_j|S_i) = (n-1)\cdot(h_0+h_1)\bar{\alpha} + h_1\cdot\sum_{j\neq i} E(S_j|S_i)$$

$$= (n-1)\cdot\left[(h_0+h_1)\cdot\bar{\alpha} + h_1\cdot E(\alpha|S_i)\right]$$

Using the rational expectations equilibrium concept, these beliefs must be correct in equilibrium. Thus (16) and the first order condition from (15) must be identical. From this we find h_0 and h_1, and substitute these back into the first order condition. The solution to the model is:

$$q_i^* = \frac{\bar{\alpha}}{n+1} + \frac{t}{2+(n-1)\cdot t}\cdot(S_i - \bar{\alpha}) \qquad i=1,\cdots,n \tag{17}$$

$$P(0) = \alpha - n\cdot\left[\frac{\bar{\alpha}}{n+1} + \frac{t}{2+(n-1)\cdot t}\cdot(S_i - \bar{\alpha})\right]$$

$$E(\Pi(0)) = \left[\frac{\bar{\alpha}}{n+1}\right]^2 + \frac{t\cdot((2-t)-(1-t)\cdot n)}{(2+(n-1)\cdot t)^2}\cdot\sigma_\alpha^2 \tag{18}$$

As in section 2, let k firms form a cartel. These firms are now able to collude over strategy choices and to pool their private information. The pooled signal can be written as:

$$S_c = \frac{1}{k}\cdot \sum_{i\in K} S_i \tag{19}$$

$$VAR(S_c) = \sigma_\alpha^2 + \frac{1}{k}\cdot\sigma_\epsilon^2$$

where K is the set of cartel members, and k the number of members. As k is known, S_c contains the same information as S_i and is hence a sufficient statistic for S_i. Thus the cartel observes a signal with a smaller variance than the individual follower.

With the cartel acting as a Stackelberg leader, each follower receives two signals, its private signal S_i and a public signal Q^c - the strategy choice of the cartel. First we assume that the followers do not attempt to extract information from the output choice of the cartel. The typical follower solves:

$$\max_{q_i} E\left[(\alpha - Q^c - \sum_{\substack{j\in N\setminus K\\ j\neq i}} q_j - q_i)\cdot q_i \,|S_i\right]$$

where α and q_j are random. Firm i's conjecture about how other follower firms react to their signal given in (16) is reformulated to include Q^c.

$$q_j^g = h_0 \cdot \bar{\alpha} + h_1 \cdot (S_j - \bar{\alpha}) + h_2 \cdot Q^c \tag{20}$$

Proceeding as above, we find the profit maximising output level of a typical follower conditional on Q^c as:

$$q_i = \frac{1}{n-k+1} \cdot \bar{\alpha} + \frac{t}{2+(n-k-1) \cdot t} \cdot (S_i - \bar{\alpha}) - \frac{1}{n-k+1} \cdot Q^c \tag{21}$$

The problem of the cartel is

$$\max_{Q^c} E\left[(\alpha - Q^c - \sum_{j \in N \setminus K} q_j) \cdot Q^c | S_c\right] \qquad \text{s.t. } (21)$$

The solution of the model is

$$q^c(k) = \frac{\bar{\alpha}}{2 \cdot k} + \frac{(2-t) \cdot t_c \cdot (n-k+1)}{2 \cdot k \cdot \varphi} \cdot (S_c - \bar{\alpha}) \tag{22}$$

$$q^f(k) = \frac{1}{2(n-k+1)} \cdot \bar{\alpha} + \frac{t}{\varphi} \cdot (S_i - \bar{\alpha}) - \frac{(2-t) \cdot t_c}{2 \cdot \varphi} \cdot (S_c - \bar{\alpha})$$

$$P(k) = \alpha - \frac{2 \cdot (n-k)+1}{2 \cdot (n-k+1)} \cdot \bar{\alpha} - \frac{t}{\varphi} \cdot \sum_{i \in N \setminus K} (S_i - \bar{\alpha}) - \frac{(2-t) \cdot t_c}{2 \cdot \varphi} \cdot (S_c - \bar{\alpha})$$

where $\varphi = 2 + (n-k-1) \cdot t_c$.

The relevant profit expression to consider when establishing the existence of a stable cartel is the unconditional expected profit of the two types of firms. This gives:

$$E(\Pi^c(k)) = \frac{\bar{\alpha}^2}{4(n-k+1)k} + \frac{(2-t)tt_c(n-k+1)}{2k\varphi^2} \cdot \sum_{i \in N \setminus K} E[(S_c - \bar{\alpha})(S_i - \bar{\alpha})]$$

$$- \frac{(2-t)^2 t_c^2 (n-k+1)}{4k\varphi^2} E[(S_c - \bar{\alpha})^2] + \frac{(2-t)t_c(n-k+1)}{2k\varphi} E[(S_c - \bar{\alpha}) \cdot \alpha] \tag{23}$$

$$E(\Pi^f(k)) = \frac{\bar{\alpha}^2}{4(n-k+1)^2} - \frac{t^2}{\varphi^2}\cdot E[(S_i-\bar{\alpha})^2] - \frac{t^2}{\varphi^2}\cdot E\Big[(S_i-\bar{\alpha}) \sum_{\substack{j\in N\setminus K \\ j\neq i}} (S_j-\bar{\alpha})\Big]$$

$$- \frac{(2-t)tt_c}{2\varphi^2}\cdot E[(S_i-\bar{\alpha})(S_c-\bar{\alpha})] + \frac{(2-t)tt_c}{2\varphi^2}\cdot E\Big[(S_c-\bar{\alpha}) \sum_{j\in N\setminus K} (S_j-\bar{\alpha})\Big]$$

$$+ \frac{(2-t)^2t_c^2}{4\varphi^2}\cdot E[(S_c-\bar{\alpha})^2] + \frac{t}{\varphi}\cdot E[(S_i-\bar{\alpha})\alpha] - \frac{(2-t)t_c}{2\varphi}\cdot E[(S_c-\bar{\alpha})\alpha] \quad (24)$$

Using $E((S_i-\bar{\alpha})^2) = VAR(S_i)$; $E((S_c-\bar{\alpha})^2) = VAR(S_c)$; $E((S_i-\bar{\alpha})\alpha) = E((S_c-\bar{\alpha})\alpha) = \sigma_\alpha^2$ and $t_c\cdot VAR(S_c) = \sigma_\alpha^2$ we can write (23) and (24) as:

$$E(\Pi^c(k)) = \frac{\bar{\alpha}^2}{4(n-k+1)k} + \frac{(2-t)^2t_c(n-k+1)}{4k\varphi^2}\cdot\sigma_\alpha^2 \quad (25)$$

$$E(\Pi^f(k)) = \frac{\bar{\alpha}^2}{4(n-k+1)^2} + \frac{4t - (2-t)(2+t)t_c}{4\cdot\varphi^2}\cdot\sigma_\alpha^2 \quad (26)$$

Note that the first term in both (25) and (26) corresponds to cartel and fringe profits under certainty, compare with equation (3) and (4). As $0\leq t\leq 1$ and $0\leq t_c\leq 1$, the second term in (25) is bounded from above by $\frac{n-k+1}{4\cdot k}\cdot\sigma_\alpha^2$. Similarly the second term in (26) is bounded from above by $\frac{1}{n-k+1}\cdot\sigma_\alpha^2$. From this follows that if σ_α^2 is not too large relative to $\bar{\alpha}$ then a stable cartel exists.

From (22) we see that the followers are able to infer the information of the cartel from Q^c. We turn to this in the next section.

4. Full information transmission from cartel to fringe.

Assume that the firms initially only observe one signal given by (12). In the present set up the cartel has no chance of obtaining any further information. As the fringe firms choose their strategies after the cartel, they could potentially obtain information from the leaders strategy choice. Assume that the follower only uses their own private information. Their gain from obtaining more information is:

$$\frac{\partial E(\Pi^f(k))}{\partial t} = \frac{(2+tt_c)+(n-k-1)\cdot(2t_c-t)}{\varphi^3}\cdot\sigma_\alpha^2 > 0 \tag{27}$$

Thus the fringe firms have an incentive to attempt to learn the information of the cartel. Turning to the cartel,

$$\frac{\partial E(\Pi^c(k))}{\partial t} = -\frac{(2-t)t_c(n-k)(n-k-1)}{\varphi^3}\cdot\sigma_\alpha^2 < 0 \tag{28}$$

$$\frac{\partial E(\Pi^c(k))}{\partial t_c} = \frac{(2-t)^2(n-k-1)}{4\cdot k\cdot\varphi^3}\cdot\sigma_\alpha^2 > 0$$

The leader prefers the signal of the followers to be noisy rather than perfect, and would, if possible, like to prevent the follower from learning from its action.[4)]

If the cartel does not recognise its ability to affect what the followers infer from the output choice of the cartel, then in the present set up there is nothing which prevents the follower from learning all the information of the leader. From the output choice of the cartel, equation (22) we see that given a follower has sufficient information to compute its functional form, the choice of cartel output is an invertible function of S_c and would fully reveal the information of the leader to the follower. The follower thus knows S_c and uses this to update his signal.[5)] Asssume that the leader does not attempt to bias its output. Then both S_i and S_c are unbiased and the only effect is on the variance. Denote the updated signal S_i^f,

$$S_i^f = \alpha + \frac{1}{k+1} \cdot \left[\sum_{j \in K} \epsilon_j + \epsilon_i \right] \tag{29}$$

$$E(S_i^f) = E(\alpha) = \bar{\alpha} \qquad \text{and} \qquad VAR(S_i^f) = \sigma_\alpha^2 + \frac{1}{k+1} \cdot \sigma_\epsilon^2$$

Denoting by t_f the precision of the signal of the follower, this can be written as:

$$t_f(k) = \frac{(k+1)\sigma_\alpha^2}{(k+1)\sigma_\alpha^2 + \sigma_\epsilon^2} \tag{30}$$

Using (29) to substitute S_i^f for S_i and (30) to substitute $t_f(k)$ for t in the equilibrium profit functions (23) and (24) and simplify using $E\left[(S_i^f - \bar{\alpha})(S_j^f - \bar{\alpha})\right] = \frac{\sigma_\alpha^2}{(k+1)^2} \cdot \left[\frac{k^2}{t_c} + 2k + 1\right]$, profits become:

$$E(\Pi^c(k)) = \frac{\bar{\alpha}^2}{4(n-k+1)k}$$

$$+ \frac{(2-t_f)(n-k+1)\left[(4t_f t_c + 2t_f(1+t_c)k)(n-k) + (2-t_f)t_c(k+1)\right]}{4 \cdot k \cdot (k+1) \cdot \varphi^2} \cdot \sigma_\alpha^2 \tag{31}$$

$$E(\Pi^f(k)) = \frac{\bar{\alpha}^2}{4(n-k+1)^2} + \frac{4t_f-4t_c+t_f^2t_c}{4\cdot\varphi^2}\cdot\sigma_\alpha^2 + \frac{(n-k-1)t_f}{2(k+1)^2\varphi^2}\cdot$$

$$\left[(2-t_f)(t_c+k)(k+1)-2t_f\left[\frac{k^2}{t_c}+2k+1\right]+(k+1)^2(2t_f-2t_c+t_ft_c)\right]\cdot\sigma_\alpha^2 \qquad (32)$$

As argued above, the interest centres around the largest number of firms for which it is true that k=n is a stable cartel. Recalling that both t_f and t_c are functions of k, we compute:

$$E(\Pi^c(n)) - E(\Pi^f(n-1)) = \frac{4-n}{16n}\cdot\bar{\alpha}^2 + \frac{t_c(n)}{4n}\cdot\sigma_\alpha^2$$

$$- \frac{4t_f(n-1)-4t_c(n-1)+t_f^2(n-1)t_c(n-1)}{16}\cdot\sigma_\alpha^2$$

$$= \frac{4-n}{16n}\cdot\bar{\alpha}^2 + t_c^2(n)\cdot\frac{(n-1)\left[(4-n)n\sigma_\alpha^2+4\sigma_\epsilon^2\right]}{16n^2\left[(n-1)\sigma_\alpha^2+\sigma_\epsilon^2\right]}$$

$$= \frac{4-n}{16n}\cdot\bar{\alpha}^2 + \xi_1\cdot\sigma_\alpha^2 \qquad (33)$$

The expression in (33) is certainly non-negative for $n \leq 4$ and may be positive for $n>4$. Even if the followers learn all the information of the cartel, there is a bias towards a larger n^*.

One could question the use of the case of perfect information, i.e. the results in section 2, as the relevant benchmark because uncertainty on its own might be enough to lead to a larger cartel. An alternative benchmark is the case where all firms shared all the available information. Putting $T = t_c = t$ in (25) and (26) we get:

$$E(\Pi^c(n)) - E(\Pi^f(n-1)) = \frac{4-n}{16n}\cdot\bar{\alpha}^2 + \frac{T(4-nT^2)}{16n}\cdot\sigma_\alpha^2$$

$$= \frac{4-n}{16n}\cdot\bar{\alpha}^2 + \xi_2\cdot\sigma_\alpha^2 \qquad (34)$$

Now $T = \frac{n\sigma_\alpha^2}{n\sigma_\alpha^2+\sigma_\epsilon^2}$. Compairing ξ_1 and ξ_2 we find that for $\xi_2>0$, $\xi_2 > \xi_1$ if:

$$(n-1)n\left[4-nT^2-(4-n)T\right]\cdot\sigma_\alpha^2 + \left[n(4-nT^2)+4T(n-1)\right]\cdot\sigma_\epsilon^2 > 0$$

which is true for $\xi_2>0$. Also $\xi_2=0 \Rightarrow \xi_1<0$. Hence the existence of differential information is on its own sufficient to bias the results towards a higher n^*, and the bias is larger than in the case where the followers learn. This indicates that when the followers can learn all the information of the cartel, making them the better informed agents, private information produce a disincentive to cartel formation.

The assumption that the cartel does not take into consideration that its output choice affects the information of the follower is made for tractability. It is a strong assumption which may seem to lead to a too small cartel by placing it at a disadvantage. This is though not necessarily the case.

Gal-Or(1987) has analysed a model with one Stackelberg leader and one follower, both having access to some private information. The leader has an incentive to under-produce in order to signal to the follower that demand is low. The follower realise this and attempt to take any induced bias into consideration. Gal-Or shows that a fully revealing equilibrium exist.6) Furthermore, the follower has the higher expected profits, a result which runs counter to the case of no uncertainty where the leader has the higher profit. The result arise because the leader is unsuccesful in misleading the follower and is thus just left with a lower output level. Further the leader has to choose a low output level as any higher output level would lead the follower to infer that demand was high and hence to expand output. Thus the leader is caught in a bad situation where he is made worse off by his private information.

If the results of Gal-Or carries over to our model, then the cartel should be even worse off if we allowed it to try and influence what the followers infer from its output choice. Thus if it was possible to solve the general case we would expect the result that private information may lead to smaller cartels to be strengthened.

It is worth noting that it is the simplicity of the present model which enables the followers to learn. It would therefore be interesting to consider cases where this is not possible (e.g. when costs are random). To this we turn in the next section.

5. Different information structures.

Assume that the model in section 2 is enlarged to include a random constant cost term $c_i \cdot q_i = \bar{c} \cdot q_i + \eta_i \, q_i$ where $\bar{c}$ is subsumed in $\bar{\alpha}$. Assume further that each firm i observes a perfect signal on their own costs $\eta_i \sim N(0, \sigma_\eta^2)$, but observe no signal on other firms costs, and that the η_i's are uncorrelated. We then get an extra source of noise in the output decision of the cartel.

The output decision of the cartel depends on the lovest costs of any member η_c. This firm will produce the total amount of cartel output. The problem of profit sharing within the cartel is presently ignored.

$$\eta_c = \min\{\eta_i \mid i \in K\}$$

The cartel has no prior knowledge on η_j, $j \in N \setminus K$ other than its distribution. Taking expectations, this nets out when writing down the first order condition of the cartel. Rewriting (22) to get this:

$$Q^c(k) = \frac{\bar{\alpha}}{2} + \frac{(2-t) \cdot t_c \cdot (n-k+1)}{2 \cdot k \cdot (2+(n-k-1) \cdot t)}(S_c - \bar{\alpha}) - \frac{n-k+1}{2} \cdot \eta_c \qquad (22')$$

from which it is not possible to infer S_c perfectly by observing $Q^c(k)$.

Rather than solving this model explicitly, we consider a general case in which the followers are not able to infer all of the cartel's information. This is done by choosing specific values of t and t_c in (25) and (26).

Assume that the information of a firm can be summarised in one signal. Further assume that this signal is identical for firms within the cartel and the group of followers, but that it may differ between these two groups. One way of achieving this would be by introducing more noise into the system as in the example above.

As we do not specify the extend of information transmission, both t_f and t_c are possibly functions of k. We are only concerned with the case where k=n is the stable cartel. For simplicity write $t_c(n)$, $t_c(n-1)$ and $t_f(n-1)$ as t_c, $t_c^{\prime}$ and $t_f^{\prime}$ respectively.

$$E(\Pi^c(n)) - E(\Pi^f(n-1)) = \frac{4-n}{16n}\cdot\bar{\alpha}^2 + \frac{t_c}{4n}\cdot\sigma_\alpha^2 - \frac{4t_f^{\prime}-(4-t_f^{\prime 2})t_c^{\prime}}{16}\cdot\sigma_\alpha^2$$

$$= \frac{4-n}{16n}\cdot\bar{\alpha}^2 + \xi_3 \qquad (35)$$

Comparing (35) with (34) for $\xi_2>0$, we find that $\xi_3 > \xi_2$ (i.e. information sharing implies a larger cartel) implies:

$$n\cdot\left[T^3 - t_c^{\prime}t_f^{\prime 2} + 4(t_c^{\prime} - t_f^{\prime})\right] - 4(T - t_c^{\prime}) \geq 0$$

Thus a necessary condition for $\xi_3 \geq \xi_2$ is that $t_c^{\prime} > t_f^{\prime}$. This implies that for information sharing to have a positive effect on the size of a stable cartel, the follower must in equilibrium be less well informed than the cartel.

Finally consider the extreme where $t = t_c = 1$. In this case (25) and (26) become:

$$E(\Pi^c(k)) = \frac{1}{4(n-k+1)k} \cdot (\bar{\alpha}^2 + \sigma_\alpha^2)$$

$$E(\Pi^f(k)) = \frac{1}{4(n-k+1)^2} \cdot (\bar{\alpha}^2 + \sigma_\alpha^2)$$

The analysis in section 2 carries through and (8) gives the unique stable cartel. Further:

$$E(\Pi^c(n)) - E(\Pi^f(n-1)) = \frac{4-n}{16n} \cdot \left[\bar{\alpha}^2 + \sigma_\alpha^2\right]$$

Thus in case of uncertainty where this is resolved prior to forming the cartel $n^* = 4$ as under certainty.

6. Related literature.

Sakai(1984) consider the role of information in a Stackelberg duopoly model. Two papers allow for collusion. Clarke(1982) considers uncertain demand and mergers in a model where all decision makers act as Cournot players. Shapiro(1985) considers trade-associations in a model with uncertain costs.

Sakai(1984) consider a case with two firms, a follower and a leader, corresponding to n=2, k=1. The two firms either observe a perfect signal, $\sigma_{\epsilon}^{2} = 0$ or a useless signal, $\sigma_{\epsilon}^{2} = \infty$. This gives rise to four possible cases. The model has the same demand structure as above, but assumes linear marginal costs. As in the first model in section 3, the follower is not allowed to learn from the leaders choice of action. We are thus able to get the results of Sakai(1984b) as special cases of the model in section 3.

In our model, set n=2, k=1, $t \in \{0,1\}$ and $t_c \in \{0,1\}$. Thus $t_c=1$, $t=1$ corresponds to the case where both firms observe a perfect signal, $t_c=0$, $t=0$ corresponds to the case where neither observes a (useful) signal, and $t_c=1$, $t=0$; $t_c=0$, $t=1$ corresponds to the mixed cases where one of the two firms is informed, the other not. We use the notation in Sakai(1984) such that $E\Pi^1(i,j)$ denotes the expected profit of firm 1 if $t_c=i$ and $t=j$, and the value pertaining to the cartel is written first. Thus $E\Pi^f(0,1)$ is the profit of the follower when $t_c=0$ and $t_f=1$. In his theorem 2, Sakai shows that:

$$E\Pi^c(0,0) = E\Pi^c(0,1) < E\Pi^c(1,1) < E\Pi^c(1,0)$$

$$E\Pi^f(1,0) < E\Pi^f(0,0) < E\Pi^f(1,1) < E\Pi^f(0,1)$$

Thus as in (25) and (26) the leader prefers the follower to be ignorant, the follower prefers to be informed. This implies that the follower would attempt to obtain extra information, and this should be allowed for in the model.

Further from the model it is easy to show that both firms would prefer to be the leader regardless of the information structure. This means that we are back to the well known problem of how to establish an endogenous leader, a problem first pointed out by Stackelberg(1934,1938).

Clarke(1982) extends a model of merger by Salant et al(1983) to a stochastic environment where firms observe a noisy signal on a random variable. By merging the firms can share their private information. The game played is a Cournot-Nash game. Even following a merger the firms choose their strategies simultaneously. This avoids any problems to do with learning and information transmission because all output levels are chosen simultaneously. The use of the Cournot-Nash solution concept implies that unless there are scale gains from merger via improved costs or information, the sole effect of the merger is to reduce the number of strategic players. Not surprising this gives rise to difficulties in obtaining stable mergers in the absense of such scale gains, because the benefit from the reduction on the number of strategic players is far greater for the non-merging firms.

With information sharing a scale gain is introduced and this does to some extend alleviate the problem. All strategic players are identical save the accuracy of their information. The merger process leading to an endogenous equilibrium coalition structure is modeled as a set of repeated static decisions by pairs of firms. Thus there is a series of discrete choices whereby the number of strategic players is reduced by one in each stage until no more mergers take place. Two criteria are considered, the "marginal" where the firms only consider the total gain from merging allowing one firms loss to be offset against the others gain, and the "average" where each firms has to gain from the merger. The former allows for side-payments, the latter not. The first problem is that the procedure together with either of the criteria prevents firms from comparing present profit with the profit ensuing from say a four firm merger. This may well be profitable even if the three mergers necessary to obtain this are not. Secondly, the procedure together with the criteria biases the results towards too few mergers as does the chosen solution concept.

Shapiro(1985) considers cost uncertainty and compare two cases "no information sharing" and "full information sharing". The game evolves in four stages: (1) the firms have the opportunity to make agreements regarding the sharing of cost information, (2) each firm i observes its own costs, (3) the firms exchange information according to any agreements in place from (1), and (4) each firm chooses an output level. It is shown that information sharing is the unique dominant strategy both under differential information and asymmetric information. Further a case where firms may join a trade-association and share its information with the other members is considered and it is shown that all firms would belong to the association. Members of the trade-association only share information. The model differs from ours in considering perfect signals and by concerning uncertainty about a "private" rather than a "public" variable.

7. Concluding remarks.

We have attempted to investigate the effect of private information when firms are colluding over strategies. If private information gives a higher incentive to collude, we have an indication of firms having an incentive to share information in equilibrium if they can also collude. To model this, the endogenous formation of a cartel which shares information and collude over strategies was considered.

Our main finding was: Differential information is not necessarily in itself sufficient to provide incentives for cartel formation. Incentives were greater if the non-cartel members were unable to learn all the information of the cartel. Thus if the cartel cannot avoid disseminating most of its information, then private information can have a detrimental effect on the size of a collusive arrangement. This indicates that the result found in the literature on information sharing (e.g. Clarke(1983b)) that firms do not share information unless they can collude over strategies may not be generally true. It will be necessary to model the simultaneous incentives to collude and share information explicitly.

A number of issues have been left aside. First, the timing of events could be questioned. Especially two questions are of interest. What would happen if firm i decided wheter or not to join a cartel after it had recieved its signal ? What if a firm could leave the cartel after the information was shared but before a strategy was chosen ? Regarding the first question, the interest centres around who will be in the cartel; those with the most extreme signal, those with the most positive (negative) signal, or those with the least extreme signal (i.e. closest to $\bar{\alpha}$). In this context firms are different ex ante to joining the cartel. This gives rise to complicated problems of how to split the joint cartel profit and of whether members are willing to reveal their information truthfully. Roberts(1985) considers a model of cartel behaviour with adverse selection, where firms observes a perfect signal before forming a cartel. In the model firms bargain over the split of profits. As bargaining power is related to the private information of the firms, full revelation of information within the cartel may not take place.

Regarding the second question, the answer is that was this possible, all firms would want to first join the cartel, get the information and then leave again, casting further doubt on solutions where is not the case that all firms are in the stable cartel. For the present one could assume that the strategy was choosen by an external coordination body (some models of information sharing consider trade associations) and cartel members were only told what to produce.[7)]

We have mainly concentrated on the case where all firms in the industry would be in a stable cartel. This is motivated by the lack of method by which a partial cartel would arise, because when profit of a cartel firm is increasing in the number of cartel members, follower firm profit exceeds cartel member profit. Allowing side-payments changes this. As total profits when a cartel exists is higher than in the Cournot equilibrium (compare (18) and (25), noting that (25) is a lower bound on per firm profits under cartellisation) there is some scope for this. When side-payments are allowed, the model does, though, take a different character, regardless of whether firms learn their information before or after joining a cartel. The problem becomes whether a mechanism can be designed such that all firms cooperate and reveal their private information fully. On the revelation principle with side-payments in a model of cartel behaviour, see Roberts(1985).

For future research, it would be of interest to consider a more dynamic structure in which firms learn about each others information more gradually. In such a case the Stackelberg solution concept may prove less usefull. In the present case, one could see the model as one in which there is a new and uncorrelated shock to demand in each period. Going back to the alternative interpretation of the game in section 2.1, one might have that information within the cartel was revealed bit by bit in the bargaining stage.

Footnotes.

1) If we assume that followers choose their strategy simultaneously, acting as Cournot-Nash players is the best they can do.

2) <x> denotes the integer part of x.

3) Alternatively (13) could have been assumed, and the the distribution in (11) and (12) chosen such that they were consistent with (13). As shown in Ericson(1969) and used in Li(1985), (13) is true for other distributions than the normal.

4) Summing (27) and (28) we get that, for given t, t_c, this is only positive for k close to n (and always positive if $k \geq n-2$). Thus selling of information would in some cases be profitable.

5) It is worth noting the difference between full, partial and no revelation. Write the output of the cartel as a function of its observed joint singal as $Q_c(S_c)$. If $S_c=Q_c^{-1}$ exists, we say that S_c is fully revealed by Q_c. If the set $\Sigma = \{S_c | Q_c=Q_c(S_c)\}$, i.e. the set of signals which could have given rise to the observed output level, is not a singleton, observing Q_c does not fully reveal S_c. If Σ is a proper subset of the set of all possible singals, observing Q_c does contain some information about S_c, and we say that Q_c is partially revealing. This information can be valuable if e.g. Σ is a closed subset of $\mathbb{R}$. If either Σ is identical to the set of possible signals or if Q_c is not a function of S_c (e.g. a constant), we have a case of no revelation.

6) There is also equilibria which may be partially revealing. In these equilibria the leaders choice of output as a function of its signal is either discontinuous or bounded.

7) If problems of firms inferrring the joint information from this arose, one could arbitrarily assume that the cartel output was shared out by some random mechanism, unknown to the individual member.

Appendix III.A.

Let:

$$L \equiv \frac{3n + 1 - \sqrt{n^2-2n-7}}{4}$$

We want to show that for $n > 4$, L is not an integer. For $n = 5$, $L \simeq 3.29$. For $n \geq 6$ we have:

$$n-1 > \sqrt{n^2-2n-7} > n-2$$

implying

$$X \equiv \frac{2n+3}{4} > L > \frac{2n+2}{4} \equiv Y$$

Now $X - Y = \frac{1}{4}$. Also either Y is an integer or $Y + \frac{1}{2}$ is an integer. Hence L cannot be an integer.

CHAPTER IV

A Two Stage Duopoly Model With Uncertainty.

1. Introduction.

Recently oligopoly models incorporating long and short run aspects have been the focus of some attention. These have been two stage games where firms in the first stage choose capacity or short run cost functions, corresponding to the notion that capital is commited prior to production. In the second stage firms firms have chosen either prices or quantities.

The first and most striking result for this type of models was Kreps and Scheinkman(1983), who shoved that the unique Nash equilibrium in a model where firms first choose capacity and then prices, correspond to the equilibrium in the one-shot Cournot oligopoly model. This was seen as a strong defence of the use of the Cournot oligopoly model, as well as offering a reinterpretation.

Later Dixon(1985,1986a proposition 4b) and Vives(1986) have generalised the model, allowing for some flexibility of production in the second stage. They show that results ranging from Cournot to Bertrand (Nash in prices) are possible, depending on the ease with which capacity can be extended in the second stage. Further Davidson and Deneckere(1986) have shovn that the result hinges critically on the assumed rationing scheme, that is, the way in which consumers are rationed when demand exceeds supply at the lowest price. Davidson and Deneckere(1986) show that with any other rationing scheme than the one used in Kreps and Scheinkman(1983), no pure strategy equilibrium exists.

We are going to extend the model by introducing uncertainty on the demand side. This is done for two reasons. Firstly to test the robustness of the models. Secondly, the model is especially interesting if one wishes to consider information transmission as it allows both voluntary and involuntary information transmission. In order to consider private information, the model has to be solved under uncertainty. As this throws up some interesting results we have chosen to present these seperatly. Private information is considered in chapter V below.

The main result of the paper is that no pure strategy equilibrium exists. This is true regardless of whether the realisation of the stochastic parameter is known after the first stage or after the second stage. Secondly a mixed strategy equilibrium exists. As an example, we characterise the equilibrium when the stochastic parameter has an uniform distribution.

The paper is organised as follows. The model is described in section 2. In section 3 the model is solved under the assumption that the realisation of the random variable is not known untill after the second stage. In section 4 the model is solved under the assumption that the realisation is known prior to choosing prices. Section 5 contains an example where the random variable is assumed to have a uniform distribution. Section 6 concludes the paper.

2. The model.

We are considering a two stage duopoly model. In the first stage each firm simultaneously chooses a capacity level k_i $(i=1,2)$. Firm i can produce up to the capacity level at constant costs, normalised to zero. The cost of producing beyond is assumed prohibitively high. The costs of installing capacity k_i is $b(k_i)$, where b is assumed convex and increasing. After the first stage, the capacities of the two firms are made known to all.

In the second stage capacities are fixed. Firms simultaneously choose a price at which they are willing to supply up to their capacity limits. Demand is given by:

$$P = \alpha - Q = \alpha - q_1 - q_2 \tag{1}$$

where P is price, q_i output of firm i and α is a random variable having density function $f(\alpha)$ with support $[\alpha_\ell, \alpha_u]$.

As total demand at the lowest price cannot always be met, we have to choose a rationing rule. For a very thorough treatment of contingent demand, see Dixon(1987). In the oligopoly literature two different rules have been employed. The first rule, which we following Dixon(1987) denote rule CCD (compensated contingent demand)[1), is described in Levitan and Shubik(1972) and used in Krebs and Scheinkman(1983), Gelman and Salop(1983), Dixon(1984) and Osborne and Pitchik(1986). It says that demand facing firm i when firm j has capacity k_j is

$$q_i^d(P_i) = \begin{cases} \alpha - P_i & P_i < P_j \\ \delta_i(\alpha-P_i) & P_i = P_j \\ \max\{0, \alpha-P_i-k_j\} & P_i > P_j \end{cases} \qquad (2)$$

where δ_i ($0 \leq \delta_i \leq 1$, $\delta_1+\delta_2=1$) is either 1/2 assuming that consumers cannot tell which firm has the larger capacity and hence choose store at random, or $\delta_i = \frac{k_i}{k_1+k_2}$. The latter is more convenient to work with as it gives fewer contingencies when $P_i=P_2$ and $k_1 \neq k_2$. The rationing rule corresponds to moving the vertical axis in a usual demand diagram to the point where output equals the capacity of firm j. In figure 1 below $q_i^d(P_i)$ is the heavy drawn line.

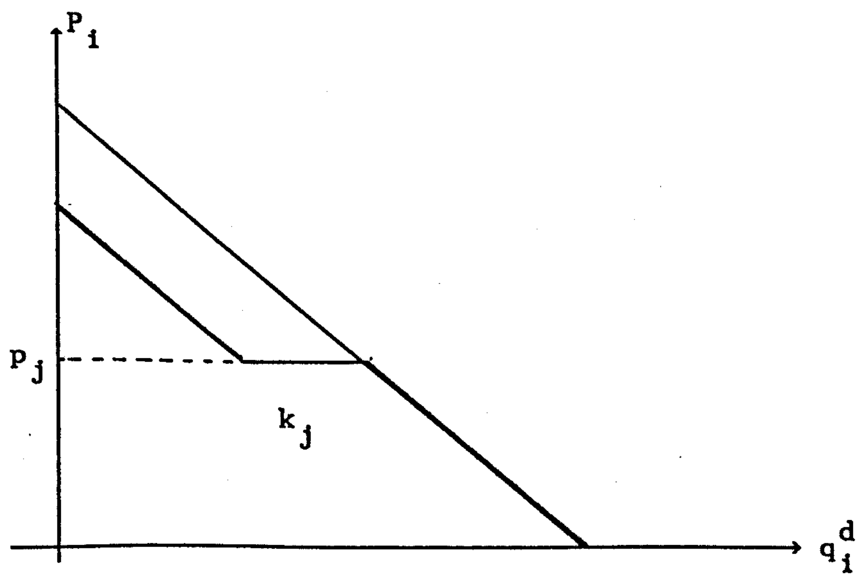

figure 1

Note that we assume that there are no income effects, as these would "twist" the demand curve. This rationing rule is the simplest to work with and can be given two interpretations, depending on the assumption about how aggregate demand is formed. If there is a large number of identical consumers, the rule allows each to buy the same fraction of the capacity k_i. This is the "limit two per customer" rule. Alternatively, if there is a large number of heterogeneous consumers, each demanding one unit of the good if the price is below their reservation price, it corresponds to assuming that the order in which consumers arrive at the queue depends positively on their reservation price.

The other rationing rule, which we following Dixon(1987) denote ED (Edgeworthian demand), originates from Edgeworth(1925)[2]. Independent of whether consumers are homogeneous or heterogeneous, a random selection is allowed to purchase their entire demand. This is a kind of first come first served rule where the place in the queue is allotted randomly. The randomness of the place in the queue may possibly introduce a random element to residual demand. One way to make the randomness harmless is to assume that all consumers are identical. Alternatively, as noted in Dixon(1987), the contingent demnad becomes non-random if households have identical homothetic preferences and there is no marginal consumer, i.e. no consumer who can only purchase part of his demand from one firm. As we are treating contingent demand as having no randomness arising from the rationing rule, we are implicitly assuming that consumers preferences are identical and homothetic. The rule is used in Allen and Hellwig(1986), Dixon(1984), Gelman and Salop(1983) and Davidson and

Deneckere(1986). The rule can be written as:

$$q_i^d(P_i) = \begin{cases} \alpha - P_i & P_i < P_j \\ \delta_i(\alpha - P_i) & P_i = P_j \\ \max\{0, \frac{\alpha - P_i}{\alpha - P_j}(\alpha - P_j - k_j)\} & P_i > P_j \end{cases} \quad (3)$$ [3)]

We show $q_i^d(P_i)$ as the heavy drawn line in figure 2 below.

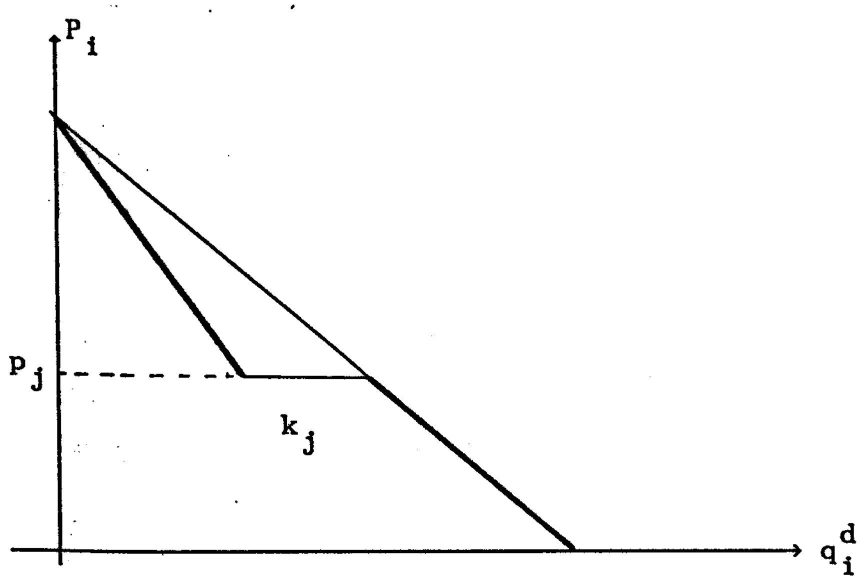

figure 2

Both rules are considered below.[4)] Note, though, that the rule CCD is much simpler to work with because the residual demand of firm i depends on the price of firm j in a very trivial manner, as it only affects the point of discontinuity of the demand function.

Throughout most of the paper, we assume that the realisation of α is made known to all firms only after the second stage. Thus both capacities and prices are chosen prior to knowing α. In section 4 below we shall, though, briefly consider a case where

α is made known after the first stage. Hence prices are chosen under full information.

Firms are assumed risk neutral and maximise expected value of profits.

3. Realisation known after stage 2.

We consider subgame perfect equilibria. The pricing stage constitutes a proper subgame, and is solved first.

3.1 The pricing subgame

In this stage capacities k_1 and k_2 are known and fixed. The strategy set A_i of firm i is

$$A_i = \{ P_i \in \mathbb{R}_+ \mid P_i \leq \alpha_u \} \qquad i=1,2$$

Given rationing scheme CCD profits can be written as:

$$E(\Pi_i(P_i,P_j)) = \begin{cases} \int_{\alpha_\ell}^{\hat{\alpha}_i} P_i(\alpha-P_i)f(\alpha)d\alpha + \int_{\hat{\alpha}_i}^{\alpha_u} P_i k_i f(\alpha)d\alpha & P_i<P_j \quad (4) \\ \int_{\alpha_\ell}^{\hat{\alpha}} P_i(\alpha-P_i)\frac{k_i}{k}f(\alpha)d\alpha + \int_{\hat{\alpha}}^{\alpha_u} P_i k_i f(\alpha)d\alpha & P_i=P_j \quad (5) \\ \int_{\tilde{\alpha}_i}^{\hat{\alpha}} P_i(\alpha-P_i-k_j)f(\alpha)d\alpha + \int_{\hat{\alpha}}^{\alpha_u} P_i k_i f(\alpha)d\alpha & P_i>P_j \quad (6) \end{cases}$$

where $\hat{\alpha} = P + k$, the α given P_i at which total capacity is exactly exhausted; $\hat{\alpha}_i = P_i + k_i$, the α given $P_i<P_j$ where i's capacity is exactly exhausted; $\tilde{\alpha}_i = P_i + k_j$, the α given $P_i>P_j$ where i's demand starts becomming positive; and $k=k_1+k_2$ is total capacity.

Given rationing rule ED profits can be written as:

$$E(\Pi_i(P_i,P_j)) = \begin{cases} \int_{\alpha_\ell}^{\hat{\alpha}_i} P_i(\alpha-P_i)f(\alpha)d\alpha + \int_{\hat{\alpha}_i}^{\alpha_u} P_i k_i f(\alpha)d\alpha & P_i<P_j \\ \int_{\alpha_\ell}^{\hat{\alpha}} P_i(\alpha-P_i)\frac{k_i}{k}f(\alpha)d\alpha + \int_{\hat{\alpha}}^{\alpha_u} P_i k_i f(\alpha)d\alpha & P_i=P_j \\ \int_{\hat{\alpha}_j}^{\beta} P_i\frac{\alpha-P_i}{\alpha-P_j}(\alpha-P_j-k_j)f(\alpha)d\alpha + \int_{\beta}^{\alpha_u} P_i k_i f(\alpha)d\alpha & P_i>P_j \end{cases}$$

where β is the positive root solving

$$\frac{\beta-P_i}{\beta-P_j}(\beta-P_j-k_j) = k_i$$

Bacause of the uncertain demand and the need for a rationing scheme it is better to charge a price slightly lower than the other firm as this ensures that the firm sells first. This indicates that equilibria in which firms charge the same price are unlikely to exist.

<u>Proposition 1</u>:

In the pricing stage, if

(i) $\min[k_1,k_2] \geq \alpha_u$ the unique pure strategy equilibrium is:

$$P_1 = P_2 = 0$$

(ii) $\max[k_1,k_2] < \alpha_u$ no pure strategy equilibrium exists.

Proof:

See appendix IV.A.

Let $B_i = \{ P_i \in \mathbb{R}_+ \mid P_i \leq \max(0, \alpha_\ell - k) \}$, i=1,2, the set of prices where even at the lowest realisation of α demand can be fully met. In the proof we use the following lemma:

Lemma 1.

The set of points of discontinuity of the payoff function A_i^{**} is:

$$A_i^{**} = \{ (P_i, P_j) \in (A_i \times A_j) \setminus (B_i \times B_j) \mid P_i = P_j \} \qquad i \neq j. \ i=1,2.$$

Proof:

See appendix IV.A.

Corollary 1.

$$\lim_{P_i \to P_j^-} E(\Pi_i(P_i, P_j)) > E(\Pi_i(P_j, P_j)) > \lim_{P_i \to P_j^+} E(\Pi_i(P_i, P_j))$$

$$\lim_{P_i \to P_j^-} E(\Pi_j(P_j, P_i)) < E(\Pi_j(P_j, P_j)) < \lim_{P_i \to P_j^+} E(\Pi_j(P_j, P_i))$$

Proof:

See appendix IV.A.

Thus the discontinuity occurs on the diagonal in the price space elsewhere the payoff function is continuous. Further, at the point of discontinuity, if one firms profit falls, the profit of the other will rise and vice versa. Thus the discontinuity we have is of a kind where the theorems of

Dasgupta and Maskin(1986a) for games with discontinous payoffs can be applied.

Proposition 2.

The pricing stage possesses a mixed-strategy Nash equilibrium (μ_i^*, μ_j^*).

Proof.

Apply theorem 5b in Dasgupta and Maskin(1986a) directly. □

To find the support of the mixed-strategy equilibrium (μ_i^*, μ_j^*), consider the price which firm i would pick if it accepts to set the high price (i.e. maximise either (6) or (9)). Denote this $\overline{P}_i$:

$$\overline{P}_i = \arg\max_{P_i > P_j} E(\Pi_i(P_i, P_j)) \qquad i=1,2. \tag{10}$$

From the proof of proposition 1, we know that this price is non negative. Define:

$$\overline{P} = \max \{ \overline{P}_i, \overline{P}_j \} \tag{11}$$

By construction no firm would choose a price higher than $\overline{P}$ with positive probability. Assume that $\overline{P}_i > \overline{P}_j$. Then firm j might pick a price $P_j \in [\overline{P}_j, \overline{P}_i]$ as there is a possibility that firm i picks $\overline{P}_i$ with positive probability. The same type of argument goes through for firm i. Hence $\overline{P}$ is the supremum of the support of μ_i^*, i=1,2.

Define $\underline{P}_i$ as the solution to

$$\int_{\alpha_\ell}^{\hat{\alpha}_i} \underline{P}_i(\alpha-\underline{P}_i)f(\alpha)d\alpha + \int_{\hat{\alpha}_i}^{\alpha_u} \underline{P}_i k_i f(\alpha)d\alpha = E(\Pi_i(\overline{P}_i, P_j)) \qquad (12)$$

$\underline{P}_i$ is the price at which firm i is indifferent between charging the lowest price (i.e. undercutting firm j) or charging the highest price, that is the price where firm i stops undercutting. Define:

$$\underline{P} = \max \{ \underline{P}_i, \underline{P}_j \} \qquad (13)$$

Then $\underline{P}$ is the infimum of the support of μ_i^* i=1,2 as the firm with the highest price would never set a price below $\underline{P}$ and the firm with the lower would only gain by raising its price to $\underline{P}$. Thus μ_i^* i=1,2, has support $[\underline{P}, \overline{P}]$.

<u>Proposition 3.</u>

μ_i^* i=1,2, is atomless on $]\underline{P}, \overline{P}]$.

<u>Proof:</u>

If μ_i^* has an atom at P_i then there is a positive probability of a tie at that price. But then, from corollary 1, one firm would prefer to shift some mass just below that price. □

The only possibility of an atom is at $\underline{P}$. Assume without loss of generality that $\underline{P}_i > \underline{P}_j$. Then it is possible that μ_j have a mass point at $\underline{P}$

Finally if $k_i=k_j$, we have:

Proposition 4.

If capacities are identical, the pricing stage possesses a symmetric mixed-strategy Nash equilibrium (μ^*,μ^*), and μ^* is atomless on $[\underline{P},\overline{P}]$.

Proof.

Apply theorem 6. in Dasgupta and Maskin(1986a). □

Write the mixed-strategy equilibrium as (μ_i^*,μ_j^*), where μ_i^* is a probability distribution with support $[\underline{P},\overline{P}]$. Note from (11) and (13) that $\underline{P}$ and $\overline{P}$ are both functions of k_i and k_j and furthermore:

$$\overline{P}(k_i,k_j) = \begin{cases} f_1(k_i,k_j) & \text{for } k_i \geq k_j \\ g_1(k_i,k_j) & \text{for } k_i < k_j \end{cases} \tag{14}$$

$$\underline{P}(k_i,k_j) = \begin{cases} f_2(k_i,k_j) & \text{for } k_i \geq k_j \\ g_2(k_i,k_j) & \text{for } k_i < k_j \end{cases} \tag{15}$$

where $f_1=g_1$ and $f_2=g_2$ for $k_i=k_j$.

For rationing scheme CCD, the payoff of the second stage is particularly simple because the price of the other firm only enter to determine the point of discontinuity.

$$E(\Pi_i(k_i,k_j)) = \int_{\underline{P}}^{\overline{P}} \Bigg\{ \left[\int_{\tilde{\alpha}_i}^{\hat{\alpha}} P_i(\alpha-P_i-k_j)f(\alpha)d\alpha + \int_{\hat{\alpha}}^{\alpha_u} P_i k_i f(\alpha)d\alpha \right] \int_{\underline{P}}^{P_i} d\mu_j^*$$

$$+ \left[\int_{\alpha_\ell}^{\hat{\alpha}_i} P_i(\alpha-P_i)f(\alpha)d\alpha + \int_{\hat{\alpha}_i}^{\alpha_u} P_i k_i f(\alpha)d\alpha \right] \int_{P_i}^{\overline{P}} d\mu_j^* \Bigg\} d\mu_i^* \qquad (16)$$

With rationing scheme (E) we get:

$$E(\Pi_i(k_i,k_j)) = \int_{\underline{P}}^{\overline{P}} \Bigg\{ \int_{\underline{P}}^{P_i} \left[\int_{\hat{\alpha}_j}^{\beta} P_i \cdot \frac{\alpha-P_i}{\alpha-P_j}(\alpha-P_j-k_j)f(\alpha)d\alpha + \int_{\beta}^{\alpha_u} P_i k_i f(\alpha)d\alpha \right] d\mu_j^*$$

$$+ \int_{P_i}^{\overline{P}} \left[\int_{\alpha_\ell}^{\hat{\alpha}_i} P_i(\alpha-P_i)f(\alpha)d\alpha + \int_{\hat{\alpha}_i}^{\alpha_u} P_i k_i f(\alpha)d\alpha \right] d\mu_j^* \Bigg\} d\mu_i^* \qquad (17)$$

3.2 The capacity stage.

Now it is clear from (16) and (17) above together with the convexity of the cost of capacity that profits are continous and concave in k_i,k_j, but also that at the point $k_i=k_j$ they are not differentiable. By e.g. theorem 1 reported in Dasgupta and Maskin(1986a), a pure strategy equilibrium exists in this stage.

Byond the existence of an equilibrium in the two stage game, consisting of a pure strategy choice of capacities and a mixed strategy choice of prices, the model, although simple, is too general for anything else to be said. To get some more results we turn to a specific distribution for α in section 5. Before turning to that we asses the effect of changing the information structure such that the realisation of α is known after the capacity stage.

4. Relisation known after stage 1.

One possible way to obtain a pure strategy equilibrium in the pricing stage is to assume that the realisation of α is made known after stage 1. If this is the case, our stage 2 becomes deterministic and is a simple version of Osborne and Pitchik(1986). They showed that given capacities and demand, there is a range of capacities which gives rise to a pure strategy equilibrium in the second stage. The pure strategy equilibrium arise when either the smaller of the capacties are larger than demand at zero price, (cf. proposition 1 case (i) above) or if capacity is such that the price which clears the market at full capacity is preferred to the optimal price of a firm consciously setting the higher price. In the other cases, no pure strategy equilibrium exists. It is the second case of pure strategy equilibria which are of interest here.

Osborne and Pitchik(1986) also show that in a pure strategy equilibrium of the second kind, the equilibrium price is the one which clears the market at full capacity. Thus if we are to get a pure strategy equilibrium, the solution to the second stage is

$$P^{*} = \alpha - k_1 - k_2.$$

As α is assumed known with certainty in the second stage, P^{*} is nonrandom. Expected payoff in stage 1 can given capacities be written as:

$$E(\Pi_i(k_i,k_j)) = E\Big[\ k_i(\alpha - k_i - k_j)\ \Big] \qquad i=1,2,\ i\neq j. \tag{18}$$

Maximising this with respect to k_i, we get:

$$k_i^* = \frac{1}{3}\,E(\alpha) \qquad i=1,2. \tag{19}$$

We now have to show that if k_i as given by (19) is chosen in the first stage, the price chosen in the second stage will be:

$$P^* = \alpha - \frac{2}{3}E(\alpha) \tag{20}$$

Firm i would never set a price below P^* as this would leave positive excess demand. Let firm i choose the higher price $\tilde{P}_i$. We have to show that P^* is a best reply to $\tilde{P}_i$ for all α. $\tilde{P}_i$ is the solution to:

$$\max_{P_i}\Big[\ P_i\ \min[k_i, \max(0, \alpha - P_i - k_j)]\Big] \tag{21}$$

We get:

$$\tilde{P}_i = \begin{cases} \text{indeterminate} & P_i > \alpha - k_j \\ \dfrac{\alpha - k_j}{2} & \alpha - k_j \geq P_i > \alpha - k \\ \alpha - k_i - k_j & \alpha - k \geq P_i \end{cases} \tag{22}$$

Inserting k_i^*

$$\tilde{P}_i = \begin{cases} \text{indeterminated} & P_i > \alpha - \frac{1}{3}\cdot E(\alpha) \\ \frac{\alpha}{2} - \frac{1}{6}\cdot E(\alpha) & \alpha - \frac{1}{3}\cdot E(\alpha) \geq P_i > \alpha - \frac{2}{3}\cdot E(\alpha) \\ \alpha - \frac{2}{3}\cdot E(\alpha) & \alpha - \frac{2}{3}\cdot E(\alpha) \geq P_i \end{cases} \quad (23)$$

Note that for $\alpha - \frac{1}{3}\cdot E(\alpha) \geq P_i > \alpha - \frac{2}{3}\cdot E(\alpha)$, $\tilde{P}_i$ has only got a well defined interior solution for $\alpha \in [\alpha_\ell, E(\alpha)]$.

To show that P_i^* is an equilibrium for some support $[\alpha_\ell', \alpha_u']$ we must show that

$$P_i^* \geq \tilde{P}_i(k_i, k_j) \text{ all } \alpha \in [\alpha_\ell', \alpha_u']$$

But we know that for $P_i > \alpha - \frac{2}{3}\cdot E(\alpha)$, $\tilde{P}_i > \alpha - \frac{2}{3}\cdot E(\alpha)$ if $\alpha \leq E(\alpha)$. Such a case exist i α has a positive variance. Hence, there exists an α such that $\tilde{P}_i > P_i^*$ and no pure strategy equilibrium exists.

A somewhat related model is considered in Harris and Lewis(1982). They consider a two-stage model where firms choose investment (or short-run cost function) in the first stage and output in the second stage. As in the presented model, firms are uncertain about the intercept of a common inverse demand function about which they observe a private signal. Further as in this section, firms are informed about the realisation of the random variable prior to the second stage. They showed the existence and uniqueness of a pure strategy equilibrium. This suggests that the choice of strategic variable can strongly influence the outcomes of the present model.

5. Uniform distributed random variable.

Assume that α has a uniform distribution on $[\alpha_l,\alpha_u]$. Further assume that firms choose the rationing scheme CCD. Then we can write expected profits of firm i as

$$E(\Pi_i(P_i,P_j)) = \begin{cases} P_i k_i & \text{if } P_i<P_j \quad (24) \\ P_i k_i - \frac{P_i}{2\Lambda}\cdot\frac{k_i}{k}\cdot(\alpha_\ell-k-P_i)^2 & \text{if } P_i=P_j \quad (25) \\ P_i k_i - \frac{P_i}{2\Lambda}\cdot(\alpha_\ell-k-P_i)^2 & \text{if } P_i>P_j \quad (26) \end{cases}$$

where $\Lambda = \alpha_u - \alpha_\ell$ and $k=k_1+k_2$.

From section 3 we know that a mixed strategy equilibrium exists in the second stage. Proceeding as in section 3.1 we can find the support of this. To find the supremum of the support $\overline{P}$, maximise (26) with respect to P_i. Solving we get:

$$\begin{aligned} \overline{P}_i &= \tfrac{2}{3}(\alpha_\ell - k) + \tfrac{1}{3}\sqrt{(\alpha_\ell-k)^2 + 6\Lambda\, k_i} \\ &= \tfrac{2}{3}(\alpha_\ell - k) + \tfrac{1}{3}\cdot\Phi(k_i) \qquad i=1,2 \end{aligned} \tag{27}$$

From (11) $\overline{P} = \max\{\overline{P}_i,\overline{P}_j\}$. Assume without loss of generality that $k_i \geq k_j$. Then $\overline{P} = \overline{P}_i$.

To find the infemum, inserting in (12) and solving yields:

$$\underline{P}_i = \overline{P}_i - \frac{\overline{P}_i(\alpha_\ell - k - \overline{P}_i)}{(\alpha_\ell - k - 3\overline{P}_i)}$$

Using (27) we can write this as:

$$\underline{P}_i = - \frac{2(\overline{P}_i)^2}{(\alpha_\ell - k - 3\overline{P}_i)} \geq (\alpha_\ell - k) > 0 \qquad (29)$$

From (13), the infimum is $\underline{P} = \max\{\underline{P}_i, \underline{P}_j\}$. Again, without loss of generality assume that $k_i \geq k_j$. Now

$$\frac{d\underline{P}_i}{dk_i} = \frac{\partial \underline{P}_i}{\partial k_i} + \frac{\partial \underline{P}_i}{\partial \overline{P}_i} \cdot \frac{\partial \overline{P}_i}{\partial k_i}$$

where k_j is held constant. Then the first term is zero and the third is positive. From (29):

$$\frac{\partial \underline{P}_i}{\partial \overline{P}_i} = \frac{4\overline{P}_i(3\overline{P}_i - \alpha_\ell + k) - 6\overline{P}_i}{(\alpha_\ell - k - 3\overline{P}_i)^2} = \frac{6\overline{P}_i^2 - 4\overline{P}_i(\alpha_\ell - k)}{(\alpha_\ell - k - 3\overline{P}_i)^2} > 0$$

The sign follows from $\overline{P}_i \geq (\alpha_\ell - k)$ for $\Delta > 0$. Hence $\underline{P}_i$ is increasing in k_i and it follows that $\underline{P} = \underline{P}_i$. Thus we can write the support of the mixed-strategy equilibrium as

$$\left[\frac{-2(\overline{P}_i)^2}{(\alpha_\ell - k - 3\overline{P}_i)}, \overline{P}_i \right]$$

Assume that firm j chooses mixed strategy $F(P_j)$. Expected profit of firm i from choosing pure strategy P_i is:

$$E(\Pi_i(P_i,F(P_j))) = \int_{\underline{P}}^{P_i} P_i k_i dF(P_j) + \int_{P_i}^{\overline{P}} (P_i k_i - \frac{P_i}{2\Lambda}(\alpha_\ell - k - P_i)^2) dF(P_j)$$

We must show that $F(\cdot)$ is a distribution function. Assume this to be the case. In equilibrium, the profit of firm i must be constant for all $P_i \in [\underline{P},\overline{P}]$. We can write this as:

$$\overline{\Pi}_i = P_i k_i - \frac{P_i}{2\Lambda}(\alpha_\ell - k - P_i)^2 F(P_i)$$

We require that $F(\underline{P}) = 0$ and $F(\overline{P}) = 1$ and $\frac{\partial F(\cdot)}{\partial P_i} > 0$ for $P_i \in [\underline{P},\overline{P}]$. Inserting we find:

$$F(\underline{P}) = 0 \quad \Leftrightarrow \quad \overline{\Pi}_i = \underline{P}\, k_i$$

$$F(\overline{P}) = 1 \quad \Leftrightarrow \quad \overline{\Pi}_i = \overline{P}\, k_i - \frac{\overline{P}}{2\Lambda}(\alpha_\ell - k - \overline{P})^2$$

$$= \frac{-2(\overline{P})^2}{(\alpha_\ell - k - 3\overline{P})} k_i = \underline{P}\, k_i$$

Further :

$$F(P_i) = \frac{P_i(3\overline{P} - \alpha_\ell + k) - 2\overline{P}^2}{P_i(P_i - \alpha_\ell + k)} \quad \frac{3P_i - \alpha_\ell + k}{3\overline{P} - \alpha_\ell + k}$$

$$\frac{\partial F(\cdot)}{\partial P_i} = \frac{1}{3\overline{P} - \alpha_\ell + k}$$

$$\frac{2\overline{P}^2[3P_i^2 - 2P_i(\alpha_\ell - k) + (\alpha_\ell - k)^2] - [3\overline{P} - (\alpha_\ell - k)][2P_i^2(\alpha_\ell - k)]}{P_i^2(P_i - \alpha_\ell + k)^2}$$

Define $Z(\cdot) = \overline{P}^2[3P_i^2-2P_i(\alpha_\ell-k)+(\alpha_\ell-k)^2]-[3\overline{P}-(\alpha_\ell-k)][P_i^2(\alpha_\ell-k)]$. Rewrite this as

$$Z(P_i) = [(3\overline{P}-x)\cdot\overline{P}+x^2]\cdot P_i^2 - 2\cdot\overline{P}^2\cdot x\cdot P_i + x^2\cdot\overline{P}^2$$

where $x = (\alpha_\ell-k)$. Now by inspection $Z(P_i)$ is convex, and $Z_{min} > 0$. Hence $Z(P_i)>0$, which implies that $\frac{\partial F(\cdot)}{\partial P_i} > 0$.

So $F(\cdot)$ is a distribution function, and we have found the mixed strategy of firm 2. We find the mixed strategy of firm 1 in the same way. In summary, the equilibrium mixed strategies of the second stage are:

$$F(P_i) = \frac{P_i(3\overline{P}-\alpha_\ell+k)-2\overline{P}^2}{P_i(P_i-\alpha_\ell+k)} \quad \frac{3P_i-\alpha_\ell+k}{3\overline{P}\ -\alpha_\ell+k}$$

$$G(P_i) = \frac{P_i(3\overline{P}-\alpha_\ell+k)-2\overline{P}^2}{P_i(P_i-\alpha_\ell+k)} \quad \frac{3P_i-\alpha_\ell+k}{3\overline{P}\ -\alpha_\ell+k}$$

The equilibrium payoffs are:

$$E(\overline{\Pi}_i(k_i,k_j)) = \begin{cases} \frac{2}{3}(\alpha_\ell-k)k_i + \frac{(\Phi(k_i))^3-(\alpha_\ell-k)^3}{27\cdot\Lambda} & \text{for } k_i \geq k_j \\ \\ \frac{2}{3}(\alpha_\ell-k)k_i + \frac{(\Phi(k_j))^3-(\alpha_\ell-k)^3}{27\cdot\Lambda}\cdot\frac{k_i}{k_j} & \text{for } k_i < k_j \end{cases} \tag{30}$$

This completely solves the pricing stage.

Assume constant per unit cost of installing capacity, for simplicity set equal to zero. Further assume that each firm believes that the other firms choice of capacity is unaffected by its choice, a reasonable assumption given that the capacities are chosen simultaneously. Then the problem of firm i is to maximise (30).

The corresponding first order condition for $k_i \geq k_j$ is

$$\frac{\partial \Pi_i}{\partial k_i} = \frac{2}{3}(\alpha_\ell - k) - \frac{2}{3}k_i + \frac{\left[\alpha_\ell - k\right]^2}{9\Lambda} + \left[\frac{1}{3} - \frac{\alpha_\ell - k}{9\Lambda}\right] \cdot \Phi(k_i) \qquad (31)$$

For $k_i < k_j$ we get:

$$\frac{\partial \Pi_i}{\partial k_i} = \frac{2}{3}(\alpha_\ell - k) - \frac{2}{3}k_i + \frac{\left[\alpha_\ell - k\right]^2}{9\Lambda} \cdot \frac{k_i}{k_j} - \frac{\left[\alpha_\ell - k\right]^3}{27\Lambda k_j}$$
$$+ \left[\frac{2}{9} + \frac{\left[\alpha_\ell - k\right]^2}{27\Lambda k_j} - \frac{\alpha_\ell - k}{9\Lambda}\right] \cdot \Phi(k_j) \qquad (32)$$

An equilibrium in the capacity stage is a pair k_i^*, k_j^* such that $k_i^* = r_i(k_j^*)$ and $k_j^* = r_j(k_i^*)$, where r_i, r_j are the reaction correspondences. $r_i(k_j)$ is the k_i which solves either (31) when $k_i \geq k_j$ or (32) when $k_i < k_j$ for a given k_j.

It is not possible to find explicit expressions for the correspondences from (31) and (32). Below in figure 3 we have shown the correspondences for given values of α_ℓ and α_u. Three points merit special attention. These are the three equilibria, marked a, b and c. The fourth point, d, is not an equilibrium but included in the following for completeness.

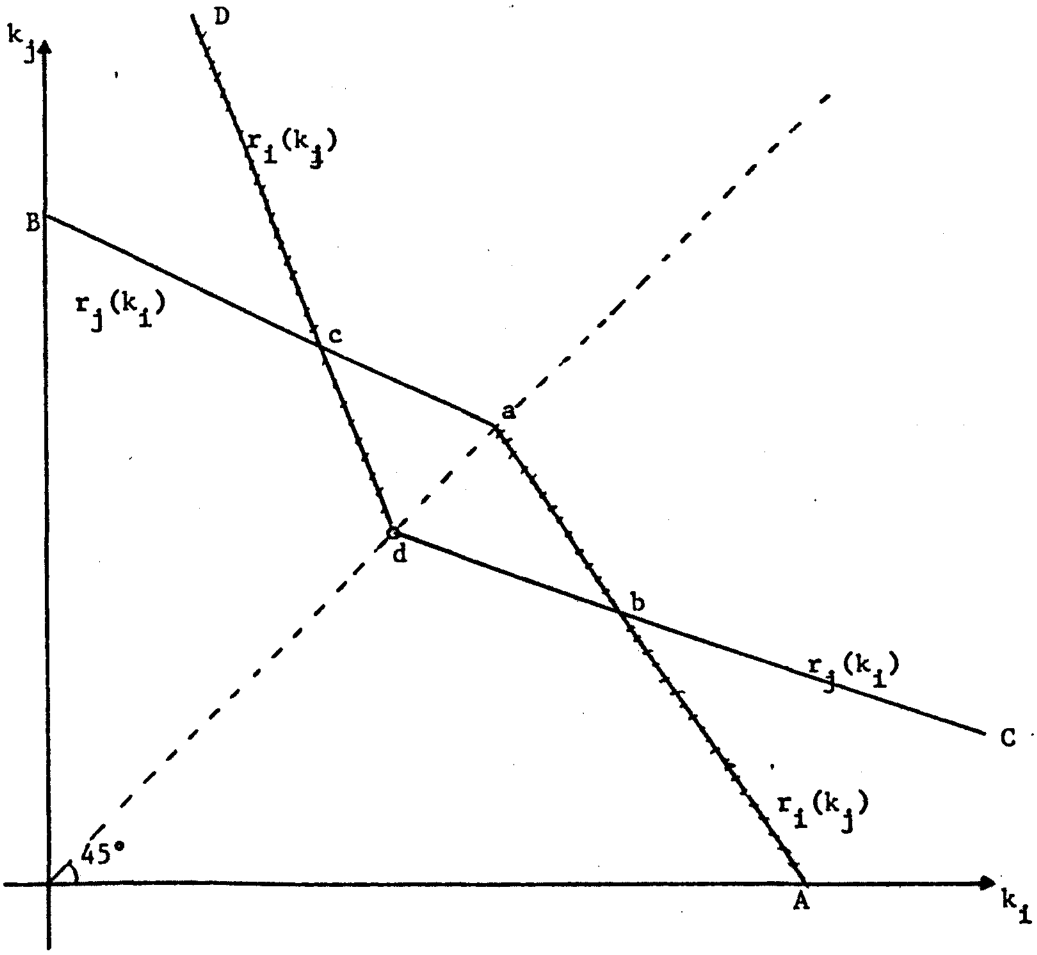

Figure 3. Reaction correspondences.

To find analytical expressions for the three equilibria, we solve for the four cases using (31) and (32) above. Although (31) and (32) are non-linear, we can find unique solutions to all four cases. These are given in table 1 below.

TABLE 1.

case	a	b	c	d
k_i	$\frac{1}{3}\cdot\alpha_\ell+\frac{1}{2}\cdot\Delta$	$\frac{1}{3}\cdot\alpha_\ell+\frac{2}{3}\cdot\Delta$	$\frac{1}{3}\cdot\alpha_\ell + \frac{1}{6}\cdot\Delta$	$\frac{1}{15}\left[\Delta+4\alpha_\ell+\sqrt{\alpha_\ell^2+8\alpha_\ell\Delta+\Delta^2}\right]$
k_j	$\frac{1}{3}\cdot\alpha_\ell+\frac{1}{2}\cdot\Delta$	$\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta$	$\frac{1}{3}\cdot\alpha_\ell+\frac{2}{3}\cdot\Delta$	$\frac{1}{15}\left[\Delta+4\alpha_\ell+\sqrt{\alpha_\ell^2+8\alpha_\ell\Delta+\Delta^2}\right]$
Π_i	$\frac{1}{9}\cdot\alpha_\ell^2$	$\left[\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta\right]^2$	$\frac{\left[\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta\right]^3}{\frac{1}{3}\cdot\alpha_\ell+\frac{2}{3}\cdot\Delta}$	$\frac{6}{15^3}\cdot\frac{\left[4\alpha_\ell+\Delta+\sqrt{\alpha_\ell+8\alpha_\ell\Delta+\Delta^2}\right]^3}{\alpha_\ell+\Delta+\sqrt{\alpha_\ell^2+8\alpha_\ell\Delta+\Delta^2}}$
Π_j	$\frac{1}{9}\cdot\alpha_\ell^2$	$\frac{\left[\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta\right]^3}{\frac{1}{3}\cdot\alpha_\ell+\frac{2}{3}\cdot\Delta}$	$\left[\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta\right]^2$	$\frac{6}{15^3}\cdot\frac{\left[4\alpha_\ell+\Delta+\sqrt{\alpha_\ell+8\alpha_\ell\Delta+\Delta^2}\right]^3}{\alpha_\ell+\Delta+\sqrt{\alpha_\ell^2+8\alpha_\ell\Delta+\Delta^2}}$

From figure 3 we can also see that b and c are stable equilibria whereas a is not. Now one can readily show that:

$$\Pi_i^b > \Pi_i^a$$

Further, if

$$4\alpha_\ell^2 - 6\alpha_\ell\Delta - \Delta^2 \geq 0 \tag{33}$$

Then

$$\Pi_i^a > \Pi_i^c$$

Now (33) holds if Δ is small relative to α_ℓ. As $\mathrm{Var}(\alpha) = \frac{1}{12}\cdot\Delta^2$, this is related to the case where the variance is sufficiently small. In the case where (33) does not hold both firms prefer an asymmetric equilibrium to the symmetric. There is though no good bases for choosing one equilibrium over another.

6. Concluding remarks.

In the two stage duopoly model proposed by Kreps and Scheinkman(1983), we have shown that under uncertainty, firms will not set the same price and hence the analogy with the Cournot outcome no longer holds. This is true regardless of whether uncertainty was resolved before or after the pricing stage. This strengthens the conclusion in Davidson and Deneckere(1986) that the result in Kreps and Scheinkman(1983) is sensitive to changes in the assumptions of the model.

The fact that we get a mixed strategy equilibrium implies that at an instant, producers of a homogeneous good almost always charge a different price. Another interesting result, which comes from our example in section 5, is that the equilibrium is possibly asymmetric with firms choosing different capacities.

As noted in section 4, Harris and Lewis(1982) showed that a pure strategy equilibrium exists if the strategic variable in the second stage is output level. This indicates that the choice af prices as strategic variables seriously complicates the model, and thus requires at least some justification.

The choice of strategic variable under uncertainty is closely related to the slope of the inverse demand curve. If it is relatively steep, the variance of demand for a given price is smaller than the variance of the price for a given output level. This suggests that if firms are concerned about the variance of profits, then the steeper the inverse demand curve, the more likely are firms to prefer to set prices.

Further in many cases firms set both the price and their capacity (e.g. newspapers, where both the price and the number of issues are printed on the frontpage). The model presented above is then the case where the capacity is committed before prices rather than simultaneously.

Footnotes.

1) The rule is sometimes referred to as reservation price rationing or parallel rationing.

2) Sometimes referred to as random rationing or proportional rationing.

3) Residual demand in this case can be written in a more familiar form. Let Q(P) be total demand at P. Then we can write residual demand as:

$$q_d^i(P_i) = Q(P_i)\cdot\left[1 - \frac{k_j}{Q(P_j)}\right]$$

This is the form given in eg. Dixon(1987, equation 1.2).

4) It could be argued that the choice of rule should be endogenised. Whether or not this is possible depends on the assumptions made about the consumers. If consumers are heterogeneous and the good is sold on a first come first served basis, the difference between the two schemes is the sequence in which consumers arrive at the queue or shop. This is exogeneous to the firm. When the consumers are homogeneous, the question of endogeneity of the rule has more merit as it is a choice between first come first served and proportional rationing, but in a sense the question is outside the present model, as we would expect the choice to depend on dynamic factors such as consumer loyality etc.

Appendix IV.A.

Proof of proposition 1, lemma 1 and corollary 1.

First we establist lemma 1, the condition under which there are discontinuities in the payoff function as given by either (4),(5) and (6) or (7), (8) and (9). The corollary follows from this proof. Finally we prove the proposition using the lemma.

Proof of lemma 1.:

Note that for $P_i \neq P_j$, $E(\Pi_i(P_i,P_j))$ is continous in P_i. Let A^{**} be the set of points of discontinuity. Then we know

$$A_i^{**} \subseteq \{(P_i,P_j) \in A_i \times A_j \mid P_i = P_j,\ i \neq j\}$$

Continuity from below requires:

$$\lim_{P_i \to P_j^-} E(\Pi_i(P_i,P_j)) = E(\Pi_i(P_j,P_j))$$

$$\int_{\alpha_\ell}^{P_j+k_i} P_j(\alpha-P_j)f(\alpha)d\alpha + \int_{P_j+k_i}^{\alpha_u} P_jk_if(\alpha)d\alpha$$

$$= \int_{\alpha_\ell}^{P_j+k} P_j(\alpha-P_j)\frac{k_j}{k}f(\alpha)d\alpha + \int_{P_j+k}^{\alpha_u} P_jk_if(\alpha)d\alpha$$

$$\Leftrightarrow$$

$$P_j=0 \quad \text{or} \quad \int_{\alpha_\ell}^{P_j+k_i} \frac{\alpha-P_j}{k_i}\cdot f(\alpha)d\alpha - \int_{\alpha_\ell}^{P_j+k_i} \frac{\alpha-P_j}{k}\cdot f(\alpha)d\alpha$$

$$+ \int_{P_j+k_i}^{P_j+k} \cdot f(\alpha)d\alpha - \int_{P_j+k_i}^{P_j+k} \frac{\alpha-P_j}{k}\cdot f(\alpha)d\alpha = 0$$

$$\Leftrightarrow$$

$$P_j=0 \quad \text{or} \quad \int_{\alpha_\ell}^{P_j+k_i} (\alpha-P_j)\cdot\frac{k_j}{k_i\cdot k_j}\cdot f(\alpha)d\alpha + \int_{P_j+k_i}^{P_j+k} \frac{k+P_j-\alpha}{k}\cdot f(\alpha)d\alpha = 0$$

As both terms are positive, the last equality can only hold if $P_j \leq a_\ell - k$. Thus for $P_j \in B_j$, the payoff function is continous from below. Note that this holds for both types of rationing schemes, as (4) and (7) and (5) and (8) are identical.

Continuity from above for scheme (S):

$$\lim_{P_i \to P_j^+} E(\Pi_i(P_i,P_j)) = E(\Pi_i(P_j,P_j))$$

$$\int_{P_j+k_j}^{P_j+k} P_j(\alpha-P_j-k_j)f(\alpha)d\alpha + \int_{P_j+k}^{\alpha_u} P_j k_i f(\alpha)d\alpha$$

$$= \int_{\alpha_\ell}^{P_j+k} P_j(\alpha-P_j)\frac{k_j}{k}f(\alpha)d\alpha + \int_{P_j+k}^{\alpha_u} P_j k_i f(\alpha)d\alpha$$

$$\Leftrightarrow$$

$$P_j=0 \quad \text{or} \quad \int_{P_j+k_j}^{P_j+k} \frac{\alpha-P_j-k_j}{k_i}\cdot f(\alpha)d\alpha - \int_{\alpha_\ell}^{P_j+k} \frac{\alpha-P_j}{k}\cdot f(\alpha)d\alpha = 0$$

$$\Leftrightarrow$$

$$P_j=0 \quad \text{or} \quad - \int_{\alpha_\ell}^{P_j+k_j} \frac{\alpha-P_j}{k}\cdot f(\alpha)d\alpha + \int_{P_j+k_i}^{P_j+k} \frac{(\alpha-P_j-k_j)\cdot k_j}{k\cdot k_j}\cdot f(\alpha)d\alpha = 0$$

As the first expression is non-negative and the second non-positive, only for $P_j=0$ or $P_j \leq \alpha_\ell - k$ does the equality hold.

Comtinuity for scheme (E):

$$\lim_{P_i \to P_j^+} E(\Pi_i(P_i,P_j)) = E(\Pi_i(P_j,P_j))$$

$$\Leftrightarrow$$

$$\int_{P_j+k_j}^{\beta} P_j \cdot \frac{\alpha-P_j}{\alpha-P_j}(\alpha-P_j-k_j)f(\alpha)d\alpha + \int_{\beta}^{\alpha_u} P_j k_i f(\alpha)d\alpha$$

$$= \int_{\alpha_\ell}^{P_j+k} P_j(\alpha-P_j)\frac{k_j}{k}f(\alpha)d\alpha + \int_{P_j+k}^{\alpha_u} P(\alpha)d\alpha$$

where β solves $\frac{\beta-P_j}{\beta-P_j}(\beta-P_j-k_j) = k_i \Leftrightarrow \beta = P_j + k$. Then we can write the above as:

$$\int_{P_j+k_j}^{P_j+k} P_j(\alpha-P_j-k_j)f(\alpha)d\alpha + \int_{P_j+k}^{\alpha_u} P_jk_if(\alpha)d\alpha$$

$$= \int_{\alpha_\ell}^{P_j+k} P_j(\alpha-P_j)\frac{k_j}{k}f(\alpha)d\alpha + \int_{P_j+k}^{\alpha_u} P_jk_if(\alpha)d\alpha$$

which corrresponds to the above case (S). From this follows that for

$P_j \in B_j$ the payoff function is continuous. Hence lemma 1 follows.

The corollary follows immediately from the above.

Proof of proposition 1.

We need to show that if $\alpha_\ell-k > 0$ and $P_j \leq \alpha_\ell-k$, firm i would choose to set $P_i > \alpha_\ell-k$ so that no equilibrium exists for $(P_i, P_j) \in (B_i \times B_j)\backslash\{(0,0)\}$. Now for $(P_i, P_j) \in B_i \times B_j$ both firms can always sell at their capacity level. Hence firm i would never set a price $P_i < \alpha_\ell-k$. Consider $P_i = \alpha_\ell-k$. If firm i contemplates increasing P_i it must be maximising either (6) or (9) as $P_i > P_j$.

Maximising (6) w.r.t. P_i we get:

$$\frac{\partial E(\Pi_i)}{\partial P_i} = k_i - k_i \int_{\alpha_\ell}^{P_j+k} f(\alpha)d\alpha + \int_{\beta}^{P_j+k} (\alpha-k_j-2\cdot P_i)\cdot f(\alpha)d\alpha$$

At $P_i = \alpha_\ell - k$, we have $\frac{\partial E(\Pi_i)}{\partial P_i} > 0$, implying that firm i wants to increase its price.

Maximising (9) w.r.t. P_i we get:

$$\frac{\partial E(\Pi_i)}{\partial P_i} = \int_{P_j+k_j}^{\beta} \frac{\alpha-2P_i}{\alpha-P_j}(\alpha-P_j-k_j)f(\alpha)d\alpha + \int_{\beta}^{\alpha_u} k_i f(\alpha)d\alpha$$

Now for $P_i = \alpha_\ell - k$,

$$\beta = \frac{\alpha_\ell - P_j}{2} + \frac{1}{2}\cdot\sqrt{(\alpha_\ell-P_j)^2 - 4\cdot(P_j k_j + (\alpha_\ell-k)(P_j+k_j)}$$

Now the term under the square root is less that $\frac{1}{2}\cdot(\alpha_\ell-P_j)$ as $P_j<\alpha_\ell-k$. Hence $\beta<\alpha_\ell$ implying that at $P_i=\alpha_\ell-k$,

$$\frac{\partial E(\Pi_i)}{\partial P_i} = k_i > 0$$

This implies that no equilibrium exists with

$$(P_i,P_j) \in (B_i \times B_j)\setminus(0,0)$$

It only remains to be shown in which cases $(P_i,P_j) = (0,0)$ as in all other cases there is a discontinuity in the payoff function and no pure strategy equilibrium exists.

Assume that $k_j < \alpha_u$. If $P_j=0$, there exists a $P_i>0$ such that $q_i^d(P_i,0) > 0$ for some $\alpha \in [\alpha_\ell,\alpha_u]$. Hence for $(P_i,P_j) = (0,0)$ to be an equilibrium we must have:

$$\min[k_i,k_j] \geq a_u$$

which is part (i) of proposition 1. This concludes the proof.

CHAPTER V

Private Information in a Two Stage Duopoly Model.

1. Introduction.

Recently oligopoly models incorporating long and short run aspects have been the focus of some attention. Kreps and Scheinkman(1983), Davidson and Deneckere(1986), Dixon(1986), Harris and Lewis(1982), Osborne and Pitchik(1986) and Vives(1986) have looked at two-stage models, where firms first choose capacity or short-run cost functions and then a price at which they are willing to trade or an output level. Apart from Harris and Lewis(1982), these models have been deterministic. The models have several nice features. They embody the idea that capacity is a long-run issue and is committed prior to any trading. Furthermore, prices rather than quantities can be strategic variables, even if the firms are producing a homogeneous product. Finally, given the sequential structure of the model, if firms possess any private information prior to choosing capacities, their choice of capacities will embody this information. By observing the capacity choices of the other firms, a firm can learn some of their information. This allows us to consider involuntary information transmission. Apart from considering information transmission, it is of considerable interest to asses the impact of uncertainty and private information on the results of these analysis. Firstly is represents a step towards greater realism. Secondly, it is important to check the roubustness of the models. Thirdly, models of imperfect competition are increasingly being used as one way to obtain a micro economic foundation of macro economics. In connection with this, the short-run - long-run models seems interesting because both investments and price formation is explicitly modelled. As uncertainty and private information is seen as an integral part of many of these micro foundation models, it is of interest to asses the impact of uncertainty.

The simplest two stage duopoly model was considered in Kreps and Scheinkman(1983). Assume that capacity is fixed upwards, i.e. once fixed, it cannot be augmented at any cost, and that the choice of capacities are made simultaneously and thereafter made publicly known. Kreps and Scheinkman(1983) showed that the only equilibrium has the two firms choosing the Cournot-quantities in the first stage and the prices associated with the Cournot duopoly model in the second.

Dixon(1985) show that if firms in the first stage choose a short-run cost function, in the second stage their output level, then depending on the flexibility of the production function and the relative price of input factors, all outcomes between Bertrand and Cournot are possible. In a less general model Vives(1986) gets the same result.

Osborne and Pitchik(1986) use parallel rationing and show under fairly general assumptions that given capacities, k_1, k_2 with $k_1 \geq k_2 > 0$ three cases will emerge. If capacity is large, a pure strategy equilibrium exists with zero profits. If capacity is in an intermediate range, mixed strategy equilibria exists. If capacity is small, a pure strategy equilibrium exists. Further they show that if the choice of capacity is endogeneous, a pure strategy equilibrium exists.

Davidson and Deneckere(1986) use a model identical to Kreps and Scheinkman(1983) apart from the rationing scheme. They show that only for the scheme chosen by Kreps and Scheinkman does one get pure strategy equilibria. For all other schemes considered, only mixed strategy equilibria exists. Furthermore these are always asymmetric and otherwise identical firms will not get the same expected payoff. This makes their equilibrium seem unreasonable.

The introduction of uncertainty into models of imperfect competition give rise to problems of intractability stemming from firms not only having to conjecture the behaviour of other firms, but also the impact of uncertainty and private information on the strategy choices of other firms. Models using model consistent (or rational) expectations have been studied in e.g. Ponssard(1979). Further there has been an interest in models of differential information, to assess the extent to which firms wish to share information. In a series of papers Clarke(1983a,b), Gal-Or(1985,1986), Li(1985), Novshek and Sonnenshein(1982), Shapiro(1985) and Vives(1984), it has been shown that firms do not generally wish to share information voluntarily. Firms may though part with their information involuntarily by their choice of strategy. This idea will be pursued here by considering a two stage duopoly model.

The effect of introducing uncertainty in the Kreps and

Scheinkman(1983) model was analysed chapter IV. It was shown that no pure strategy Nash equilibrium exists in the pricing stage, but a mixe strategy equilibrium exists.

Analysing the effect of private information in a model with mixed strategy equilibrium is too complicated. Secondly it is difficult to interpret the meaning of a mixed strategy equilibrium. Varian(1982) attempts to interpret it as random sales, but that does not seem convincing. Further as has been pointed out in Davidson and Deneckere(1986), mixed strategy equilibrium are very unstable and essentially requires the consumer to react quicker than the producer such that low price firms have sold out before they can raise their price.

To get a pure strategy equilibrium we loosely speaking assume that a firm when contemplating uncercutting by an infinitesimal amount considers the effect of the resulting caos from price cutting. This assumption is in the spirit of an assumption often made in price-setting duopoly models with no pure strategy equilibria. In location models e.g. Novshek(1980) and Archibald et al(1986) the assumption is often referred to as the no-mill-price-undercutting rule. A similar asssumption is made in Eaton and Kierzkowski(1984) and Ireland(1987).

In a related paper Harris and Lewis(1982) assumed that firms first chose investment (or short-run cost functions) then quantities. Firms observe a signal on demand prior to choosing their investment and observe the realisation of the random variable prior to choosing output levels. They show the existence and uniqueness of a pure strategy equilibrium in their model.

The paper is organised as follows. The model is described in section 2. In section 3 we show that an endogeneous high price setter exists. The firm setting the high price is the firm with the largest capacity. This implies that an endogeneous Stackelberg leader exists in the present model. This firm also earns the highest profit. Having solved the pricing stage, firms choose capacities to maximise profits assuming that the choice of the other firm is independent of their own choice. This assumption is reasonable given a simultaneous choice of capacity. We show that if the variance of the random parameter is not too large relative to the infimum of its support, a symmetric solution exists, which have firms choosing higher capacities than on average in the Cournot case. Else only asymmetric equilibria exists. To make this meaningfull we could reinterpret the model in terms of entry.

In section 4 we consider the effect of private information. Private information in this context is knowing that the random variable will lie within a subset of its support. For example, a firm may know whether the realisation of the random variable will lie in the upper or lower half of its range. In the first set of examples it is assumed that only one of the firm has any private information. We first consider a special example which has an analytical solution. Secondly we solve some numerical examples. These indicate that the better informed does not have an incentive to share its information with the worse informed.

More surpricingly, we find that there are cases where the worse informed has no incentive to get this information. Finally we consider some examples where both have access to some private information.

Section 5 concludes the paper.

2. The model.

We are considering a two stage duopoly model. In the first stage each firm simultaneously chooses a capacity level k_i $(i=1,2)$. Firms can produce up to k_i at constant per unit costs, normalised to zero. Production above the capacity limit implies prohibitively high costs. The costs of installing capacity k_i be $b(k_i)$, where b is assumed convex.

In the second stage capacities are fixed. Firms simultaneously choose a price at which they are willing to sell up to their capacity limit. Demand is given by:

$$P = \alpha - Q = \alpha - q_1 - q_2 \tag{1}$$

where P is price, q_i output of firm i and α is a random variable having an uniform distribution on $[\alpha_\ell, \alpha_u]$.

As total demand at the lowest price cannot always be met, we have to choose a rationing rule. We choose the one used in Kreps and Scheinkman(1983), often called parallel rationing. It says that demand facing firm i when firm j has capacity k_j is

$$q_i^d(P_i) = \begin{cases} \alpha - P_i & P_i < P_j \\ \delta_i(\alpha - P_i) & P_i = P_j \\ \max\{0, \alpha - P_i - k_j\} & P_i > P_j \end{cases} \tag{2}$$

where δ_i ($0 \leq \delta_i \leq 1$, $\delta_1 + \delta_2 = 1$) is either 1/2 assuming that consumers cannot tell which firm has the larger capacity and hence choose store at random, or $\delta_i = \frac{k_i}{k_1+k_2}$. The latter is more convenient to work with as it gives fewer contingencies when $P_1=P_2$ and $k_1 \neq k_2$. The rationing rule is illustrated in figur 1 below:

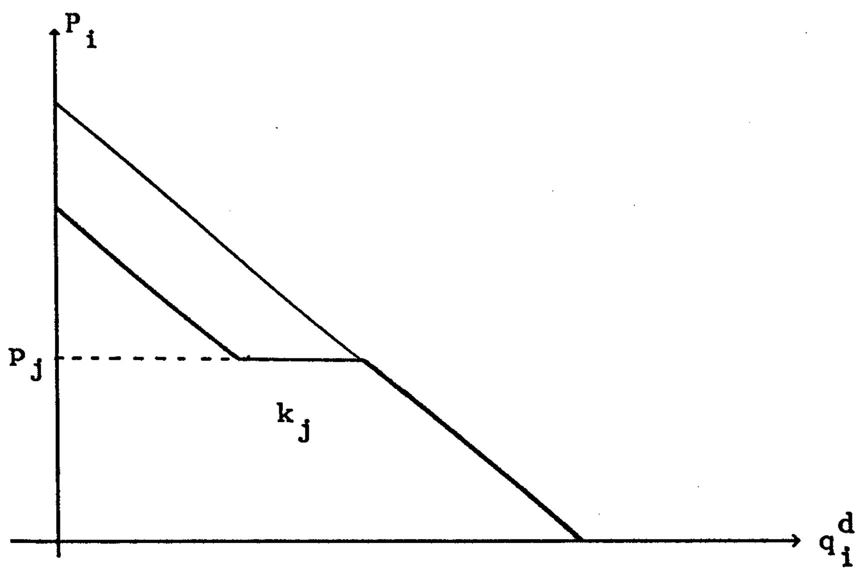

figure 1

Note that we assume that there are no income effects, as these would "twist" the demand curve. This rationing rule is the simplest to work with and can be given two interpretations, depending on the assumption about how aggregate demand is formed. If there are a large number of identical consumers, the rule allows each to buy the same fraction of the capacity k_i. This is the "limit two per customer" rule. Alternatively, if there are a large number of heterogeneous consumers, each demanding one unit of the good if the price is below their reservation price, it corresponds to assuming that the order in which consumers arrive at the queue depends positively on their reservation price[1).

Throughout the paper, we assume that the realisation of α is made known to all firms only after the second stage. Thus both capacities and prices are chosen prior to knowing α.

Firms are assumed risk neutral and maximise expected value of profits.

3. The solution to the model.

As we showed in chapter IV, no pure strategy equilibrium exist in which firms charge the same price. This result follows from uncertainty creating a mass-point of demand just below any given price for which capacity is not always fully exhausted. At $P_i=P_j$, given capacities, firms share the fluctuation in demand. Specifically, there will be realisations of demand where neither firm will be able to sell up to their capacity limit. At $P_i=P_j+\epsilon$, $\epsilon>0$, firm i sells first and will not bear the same share of a reduction of demand as when prices are equal. Firm j will bear all (or most) of the fluctuation in demand. Hence any price $P_i=P_j$ at which there are realisations of demand that do not allow firms to sell up to their capacity level will be undercut.

We need an equilibrium concept which will drive a wedge between the two prices. To this end we assume that firms conjecture the optimal response of the other firm. Thus the low price firm must choose a sufficiently low price that it does not trigger off a price war[2)]. We assume that the equilibrium outcome of a price war is predictable. Thus we do in some sense require that the conjectures are consistent.

3.1 The pricing stage.

To reduce the number of contingent cases, assume that $\min[k_1,k_2] \leq \alpha_u$ and $P_i \geq \max[o,\alpha_\ell-k]$ i=1,2. Both restrictions are included for computational ease and do not affect the results. The former requires that firms in a price-setting duopoly never finds it optimal to have a capacity so large that the only equilibrium price is zero, yielding zero profits. The latter only restricts the firms from setting prices so low that even in the worst possible case there is general excess demand. Finally, $P_i \in [0, \alpha_u]$, i.e. prices are non-negative and for some α demand is non-negative.

Without loss of generality we concentrate on firm i. Write the profit of firm i as:

$$E(\Pi_i(P_i,P_j)) = \begin{cases} P_i k_i & P_i < P_j \\ \dfrac{1}{\alpha_\ell - \alpha_u} \displaystyle\int_{\alpha_\ell}^{P_i+k} P_i \cdot (\alpha - P_i)\frac{k_i}{k} d\alpha + \dfrac{1}{\alpha_\ell - \alpha_u} \displaystyle\int_{P_i+k}^{\alpha_u} P_i k_i d\alpha & P_i = P_j \\ \dfrac{1}{\alpha_\ell - \alpha_u} \displaystyle\int_{\alpha_\ell}^{P_i+k} P_i(\alpha - P_i - k_j) d\alpha + \dfrac{1}{\alpha_\ell - \alpha_u} \displaystyle\int_{P_i+k}^{\alpha_u} P_i k_i d\alpha & P_i > P_j \end{cases}$$

We have two cases to worry about. We can dismiss one easily. Consider the case where $P_i + k \geq \alpha_u$ Then the above reduces to:

$$E(\Pi_i(P_i,P_j)) = \begin{cases} P_i \cdot k_i & P_i < P_j \\ P_i \cdot (E(\alpha) - P_i) \cdot \dfrac{k_i}{k} & P_i = P_j \\ P_i \cdot (E(\alpha) - P_i - k_j) & P_i > P_j \end{cases}$$

Assume that firm i wants to set the high price. First order conditions become:

$$E(\alpha) - k_j - 2\cdot P_i = 0$$

This implies that $E(\Pi_i(k_i,k_j)) = \frac{1}{4}\cdot(E(\alpha)-k_j)^2$. To avoid holding excess capacity, firm i would choose k_i in the first stage to meet residual demand at P_i, i.e. $k_i = \frac{1}{2}(E(\alpha)-k_j)$. We assumed that $P_i \geq \alpha_u - k \implies \frac{1}{2}(E(\alpha)-k_j) \geq \alpha_u - k_i - k_j$. Incerting for k_i we get:

$$\frac{1}{2}\cdot(E(\alpha)-k_j) \geq \alpha_u - k_j - \frac{1}{2}\cdot(E(\alpha) - k_j)$$

which implies that $E(\alpha) \geq \alpha_u$. This is only possible for the degenerate case.

Returning to the case where $P_i < \alpha_u - k$, we get by integrating out:

$$E(\Pi_i(P_i,P_j)) = \begin{cases} P_i k_i & P_i<P_j \quad (3) \\ P_i\cdot k_i - \frac{1}{2}\cdot\frac{k_i}{k}\cdot\frac{P_i(\alpha_\ell-P_i-k)^2}{\alpha_u - \alpha_\ell} & P_i=P_j \quad (4) \\ P_i\cdot k_i - \frac{1}{2}\cdot\frac{P_i(\alpha_\ell-P_i-k)^2}{\alpha_u - \alpha_\ell} & P_i>P_j \quad (5) \end{cases}$$

Maximising (5) with respect to P_i we get the following first and second order condition.

$$2\cdot(\alpha_u-\alpha_\ell)\cdot k_i = 3\cdot P_i^2 - 4\cdot(\alpha_\ell - k)\cdot P_i + (\alpha_. - k)^2 \tag{6}$$

$$\frac{\partial^2 E(\Pi_i)}{\partial P_i^2} = -\frac{3 \cdot P_i - 2 \cdot (\alpha_\ell - k)}{\alpha_u - \alpha_\ell} \leq 0 \tag{7}$$

Solving (6) we get the following solutions:

$$P_i^* = \frac{2}{3} \cdot (\alpha_\ell - k) + \frac{1}{3}\sqrt{(\alpha_\ell - k)^2 + 6 \cdot (\alpha_u - \alpha_\ell) \cdot k_i} \tag{8}$$

$$P_i = \frac{2}{3} \cdot (\alpha_\ell - k) - \frac{1}{3}\sqrt{(\alpha_\ell - k)^2 + 6 \cdot (\alpha_u - \alpha_\ell) \cdot k_i} \tag{9}$$

Inserting (8) and (9) in (7) we find that (8) is a local maximum, (9) a local minimum. Inserting back in (5) we get firm i's optimal profits as a function of the pair of capacities.

$$E(\Pi_i(k_i, k_j)) = \frac{2}{3}(\alpha_\ell - k)k_i - \frac{\left[\alpha_\ell - k\right]^3 - \left[\sqrt{(\alpha_\ell - k)^2 + 6(\alpha_u - \alpha_\ell)k_i}\right]^3}{27 \cdot (\alpha_u - \alpha_\ell)}$$

$$= \frac{2}{3}(\alpha_\ell - k)k_i + \frac{(\varphi(k_i))^3 - (\alpha_\ell - k)^3}{27 \cdot (\alpha_u - \alpha_\ell)} \tag{10}$$

where for notational simplicity:

$$\varphi(k_i) = \sqrt{(\alpha_\ell - k)^2 + 6(\alpha_u - \alpha_\ell)k_i} \geq (\alpha_\ell - k) \tag{11}$$

To show that P_i^* given in (8) is the global maximum, note first from (7) that for $P_i > \frac{2}{3} \cdot (\alpha_\ell - k)$, $E(\Pi(P_i, P_j))$ is concave. Thus we need only compare (10) to profits at the lower bound on P_i, $P_i = 0$. From (3)-(5) $E(\Pi(0, P_j)) = 0$. It is not possible to show that (10) is generally positive. Clearly the second term is always positive so a sufficient condition is $\alpha_\ell \geq k$. When we solve the first stage we have to make shure that the capacities are such that profits given by (10) are positive.

For firm i to be willing to charge the highest price and thus take on all the risk, it must not be able to make a higher expected profit by undercutting the price of firm j slightly. If it undercuts firm j, then by our assumption it sells all its capacity. Its profit would be $(P_j - \epsilon)k_i$. Thus for an equilibrium we require $\Pi_i > (P_j - \epsilon)k_i$. The highest price of firm j compatible with this is:

$$P_j = \frac{\Pi_i(k_i,k_j)}{k_i} \tag{12}$$

Using (10) the highest firm j price compatible with an equilibrium is:

$$P_j = \frac{2}{3}(\alpha_\ell-k) + \frac{(\varphi(k_i))^3 - (\alpha_\ell-k)^3}{27\cdot(\alpha_\ell-\alpha_u)\cdot k_i} \tag{13}$$

The corresponding profit is:

$$\Pi_j(k_j,k_i) = \Pi_i(k_i,k_j)\frac{k_j}{k_i} \tag{14}$$

Using (10) we get

$$\Pi_j(k_j,k_i) = \frac{2}{3}(\alpha_\ell-k)k_j + \left[\frac{(\varphi(k_i))^3 - (\alpha_\ell-k)^3}{27\cdot(\alpha_u-\alpha_\ell)}\right]\cdot\frac{k_j}{k_i} \tag{15}$$

This solves the pricing stage.

3.2 A Stackelberg type equilibrium.

Before solving the first stage, consider a model with k_i and k_j fixed. To get an equilibrium in this case, we must show that for any k_i, k_j, one of the firms prefer to set the high price while the other prefer to set the low. This follows as we know that setting the same price is never optimal. We can interpret such an equilibrium in terms of the Stackelberg duopoly model where firms choose price sequentially. The question is then whether one of the firms is willing to commit itself to a price before the other. Such a firm would be an endogeneous Stackelberg leader. Clearly this firm would be the low price firm, as a high price would be undercut slightly by the other firm. Below we shall show that such an endogeneous price leader exists. Let superscripts h and ℓ denote high and low price firm respectively. We can show the following

Lemma 1.:

$$k_i > k_j \implies \Pi_i^h > \Pi_i^\ell > \Pi_j^\ell > \Pi_j^h$$

$$k_i = k_j \implies \Pi_i^h = \Pi_i^\ell = \Pi_j^\ell = \Pi_j^h$$

Proof:

See appendix V.A.

From lemma 1 follows that if $k_i > k_j$ firm j is an endogeneous Stackelberg leader, setting the low price. This is so as firm i has no incentive to undercut the low price of firm j, whereas firm j would have an incentive to do so. Thus given our equilibrium concept we find that the smaller firm is an endogeneous Stackelberg leader, setting the lower price and getting the smaller profit.

Finally when firms have identical capacities, two symmetric equilibria exists in which the firms are indifferent between setting the high and the low price.

Keeping the difficulties in justifying a Stackelberg leader in mind this resulty is interesting in its own right. Stackelberg(1934) points out some very special cases where an endogeneous leader exists[3)]. Endogeneous leaders have also been identified by Boyer and Moreaux(1983) and Ireland(1987).

Boyer and Moreaux(1983) consider a deterministic model with linear demand and different but constant marginal costs. Firms both announce a price and the quantity they are wiling to sell at that price. Their main results are: No pure strategy Nash equilibrium exists. If costs are very diferent, the high cost firm is driven out of the market. If costs are in an intermediate range, the low cost firm is an endogeneous Stackelberg leader. If the cost difference is low, both firms prefer to be Stackelberg followers.

Ireland(1987) considers a leadership model where firms produce a differentiated product under different costs. Demand is discountinous as part of the consumers prefer the low price good, whereas other have preferences over the good and might still buy the dearer good. Because of the mass point in demand, no pure strategy equilibrium exists, but a Stackelberg equilibrium with the low cost firm as the low price setter exists. This result hinges on the cost difference.

3.3 The capacity stage.

In section 3.1 we solved the pricing stage. Using the solution we now turn to the capacity stage. We initially assumed convex capacity costs and it is worth noting that if the fixed capacity costs from this stage is taken into account, then lemma 1 is not changed. For simplicity we consider only zero capacity costs. From (10) and (15) we can write the expected profit of firm i as a function of capacities as:

$$E(\Pi_i(k_i,k_j)) = \begin{cases} \frac{2}{3}(\alpha_\ell - k)k_i + \dfrac{(\Phi(k_i))^3 - (\alpha_\ell - k)^3}{27\Lambda} & \text{for } k_i \geq k_j \\ \\ \frac{2}{3}(\alpha_\ell - k)k_i + \dfrac{(\Phi(k_j))^3 - (\alpha_\ell - k)^3}{27\Lambda} \cdot \dfrac{k_i}{k_j} & \text{for } k_i < k_j \end{cases} \tag{16}$$

Note that $E(\Pi_i(k_i,k_j))$ is continuous everywhere in k_i but not differentiable at $k_i=k_j$. Thus the reaction correspondence defined implicitely by the first order condition for maximising (16) display a discontinuity at $k_i=k_j$. The correspondences are shown in figure 2 below.

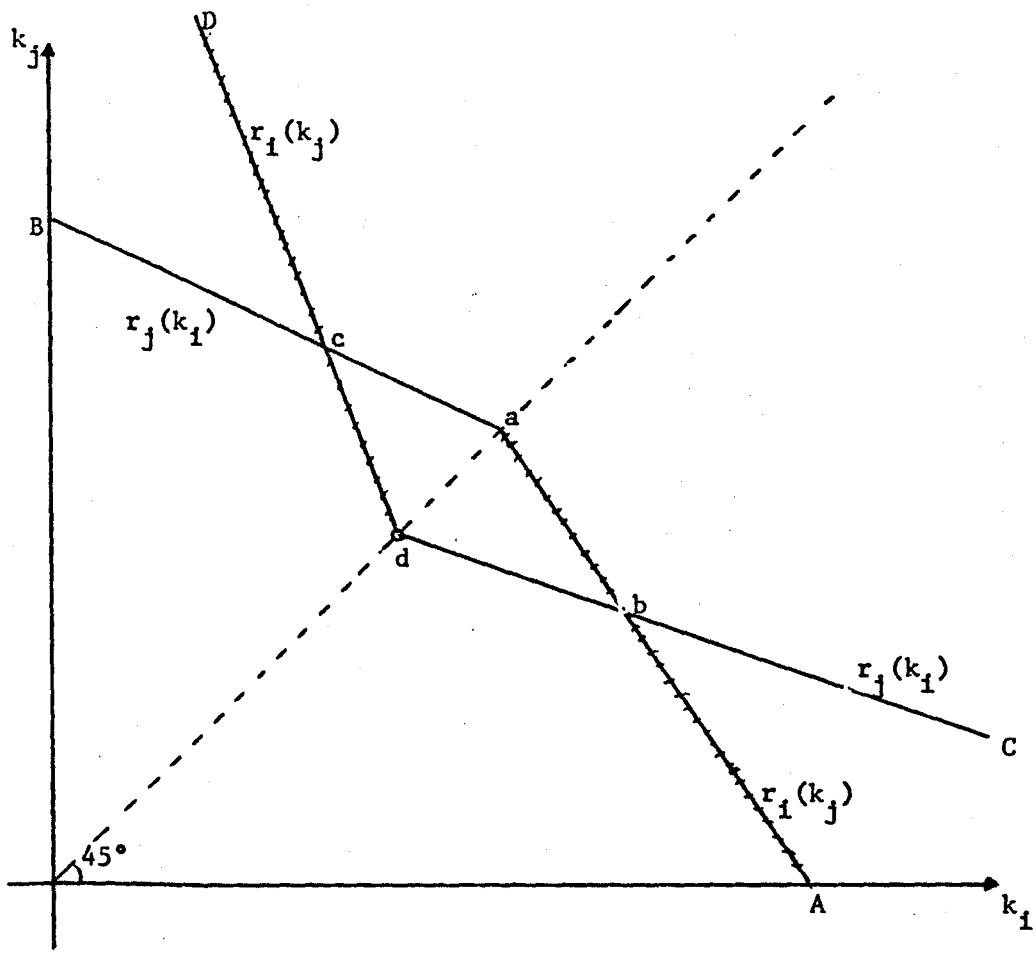

Figure 2. Reaction correspondences.

The reaction correpsondence of firm i $r(k_i)$ is given by the segments Aa and dD, where Aa comes form maximising the top expression in (16) and dD (not including the point d at which $k_i=k_j$) comes from maximising the bottom expression in (16). The corresponding segments for firm j are Ba and dC. The interest centres on the three equilibria marked a, b and c in the figure. Below we consider these (as well as the last remaining case d for completeness) in turn:

Case (a): Both firms act as if they were going to be the high capacity firm, i.e. both firm maximise (10). First order conditions are:

$$\frac{\partial \Pi_i^h}{\partial k_i} = \frac{2}{3}(\alpha_\ell - k) - \frac{2}{3}k_i + \frac{\left[\alpha_\ell - k\right]^2}{9\Delta} + \left[\frac{1}{3} - \frac{\alpha_\ell - k}{9\Delta}\right]\varphi(k_i) = 0 \quad i=1,2 \quad (17)$$

The second order condition for a maximum is:

$$\frac{\partial^2 \Pi_i^h}{\partial k_i^2} = -\frac{4}{3} - \frac{2\cdot(\alpha_\ell - k)^2 - \varphi(k_i)}{9\Delta} + \frac{(3\alpha_u - 4\alpha_\ell + k)^2}{9\Delta\varphi(k_i)} \leq 0 \qquad (18)$$

We shall only consider the symmetric solution. These are found to be:

$$k_i^* = \frac{1}{3}\cdot\alpha_\ell + \frac{1}{2}\cdot\Delta \qquad i=1,2 \qquad (19)$$

$$k_i' = \frac{1}{8}\cdot\Delta\cdot\left[1 + \sqrt{1 + 4\cdot\alpha_\ell/\Delta}\right]^2 \qquad i=1,2 \qquad (20)$$

Evaluating (18) at (19) and (20) we get:

$$\frac{\partial^2 \Pi_i^h}{\partial k_i^{*2}} = -\frac{\alpha_\ell}{27\Delta} + \frac{\frac{1}{9}\alpha_\ell^2 - 6\alpha_\ell\Delta - 15\Delta^2}{9\Delta(\alpha_\ell + 6\Delta)} < 0$$

$$\frac{\partial^2 \Pi_i^h}{\partial k_i'^2} = \left[\sqrt{1 + 4\cdot\alpha_\ell/\Delta} - 1\right]^2 > 0$$

Thus (19) is a local maximum and (20) is a local minimum. We want to show that (19) is a global maximum. Firstly, $k_i' > k_i^*$ as:

$$\frac{\Delta}{8}\cdot\left[1 + \sqrt{1 + 4\alpha_\ell/\Delta}\right]^2 = \frac{\Delta}{4} + \frac{1}{2}\alpha_\ell + \frac{\Delta}{4}\sqrt{1 + 4\alpha_\ell/\Delta} \geq \frac{1}{3}\cdot\alpha_\ell + \frac{1}{2}\cdot\Delta$$

$$\Longrightarrow$$

$$\sqrt{1 + 4\alpha_\ell/\Delta} \geq 1 - \frac{6}{4}\cdot\alpha_\ell/\Delta$$

At $k_i^\prime$, $i=1,2$, we find that $P_i^*=0$ and hence, $E(\Pi(k_i^\prime,k_j^\prime))=0$. Also, for $k_i=k_j$, $\lim_{k\to 0} E(\Pi(k_i,k_j))=0$. The final thing to demonstrate is that if $k_i=k_j$, then $P_i \geq 0 \Longrightarrow k_i \leq k_i^\prime$. From (8), note that P_i can only become negative if $k>\alpha_\ell$. Rewrite (8) as:

$$\varphi(k_i) \geq 2(2\cdot k_i - \alpha_\ell)$$

As both sides are assumed positive, we can square these without reversing the inequality sign. Squaring and rearranging we get:

$$0 \geq (2\cdot k_i - \alpha_\ell)^2 - 2\cdot\Delta\cdot k_i$$

$$= 4\cdot\left[k_i - \frac{\Delta}{4} - \frac{\alpha_\ell}{2} + \frac{\Delta}{4}\sqrt{1+4\alpha_\ell/\Delta}\right]\cdot\left[k_i - \frac{\Delta}{4} - \frac{\alpha_\ell}{2} - \frac{\Delta}{4}\sqrt{1+4\alpha_\ell/\Delta}\right]$$

$$= 4\cdot\left[k_i - \frac{\Delta}{8}\cdot\left[1 - \sqrt{1+4\alpha_\ell/\Delta}\right]^2\right]\cdot(k_i - k_i^\prime)$$

Now by assumption, $2\cdot k_i \geq \alpha_\ell$. Hence the first term is always positive, implying that for positive prices we require $k_i \leq k_i^\prime$.

From this we get that restricting attention to non-negative prices, capacities must lie in the range $[0,k_i^\prime]$, $i=1,2$. As $E(\Pi(k_i,k_j))$ attain its local maximun in the interior of this set and its local minimum on the upper bound of the set, k_i^* is the global maximum on the set $[0,k_i^\prime]$. Finally $E(\Pi(k_i^*,k_j^*)) = \frac{1}{9}\cdot\alpha_\ell^2 > 0$, so P_i^* is a global maximum.

<u>Case (b):</u> Firm j act as if it will have the low capacity while firm i acts as if it wants to have the high capacity. We get the following set of first order conditions:

$$\frac{\partial \Pi_i^h}{\partial k_i} = \frac{2}{3}(\alpha_\ell - k) - \frac{2}{3}\cdot k_i + \frac{\left[\alpha_\ell - k\right]^2}{9\Delta} + \left[\frac{1}{3} - \frac{\alpha_\ell - k}{9\Delta}\right]\cdot\varphi(k_i) = 0$$

$$\frac{\partial \Pi_j^\ell}{\partial k_j} = \frac{2}{3}(\alpha_\ell - k) - \frac{2}{3}k_j + \frac{k_j\left[\alpha_\ell - k\right]^2}{9\Delta k_i} - \frac{\left[\alpha_\ell - k\right]^3}{27\Delta k_i}$$

$$+ \left[\frac{2}{9} + \frac{\left[\alpha_\ell - k\right]^2}{27\Delta k_i} - \frac{(\alpha_\ell - k)k_j}{9\Delta k_i}\right]\varphi(k_i) = 0 \qquad (21)$$

The solution to this is:

$$k_i = \frac{1}{3}\cdot\alpha_\ell + \frac{2}{3}\cdot\Delta$$

$$k_j = \frac{1}{3}\cdot\alpha_\ell + \frac{1}{6}\cdot\Delta$$

which is a local maximum and

$$k_i = \frac{1}{8}\cdot\Delta\cdot\left[1 + \sqrt{1 + 4\cdot\alpha_\ell/\Delta}\right]^2 \qquad i=1,2$$

which is a local minimum.

Case (c), where firm i acts as if it chooses the low capacity and firm j the high follow by symmetry of the problem.

<u>Case (d):</u> Both act (inconsistently) as if they choose the low capacity, i.e. both maximise (15). First order conditions are:

$$\frac{\partial \Pi_j^{\ell}}{\partial k_j} = \frac{2}{3}(\alpha_\ell - k) - \frac{2}{3}k_j + \frac{k_j\left[\alpha_\ell - k\right]^2}{9\Delta k_i} - \frac{\left[\alpha_\ell - k\right]^3}{27\Delta k_i}$$

$$+ \left[\frac{2}{9} + \frac{\left[\alpha_\ell - k\right]^2}{27\Delta k_i} - \frac{(\alpha_\ell - k)k_j}{9\Delta k_i}\right]\varphi(k_i) = 0 \qquad i=1,2$$

The local maximum is:

$$k_i = \frac{1}{15}\cdot\left[\Delta + 4\cdot\alpha_\ell + \sqrt{\Delta^2 + 8\cdot\Delta\cdot\alpha_\ell + \alpha_\ell^2}\right] \qquad i=1,2$$

whereas the local minimum is as for the other cases.

The results are summarised in table 1. below.

Table 1.

case	a	b	c	d
k_i	$\frac{1}{3}\cdot\alpha_\ell+\frac{1}{2}\cdot\Delta$	$\frac{1}{3}\cdot\alpha_\ell+\frac{2}{3}\cdot\Delta$	$\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta$	$\frac{1}{15}\left[\Delta+4\alpha_\ell+\sqrt{\alpha_\ell^2+8\alpha_\ell\Delta+\Delta^2}\right]$
k_j	$\frac{1}{3}\cdot\alpha_\ell+\frac{1}{2}\cdot\Delta$	$\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta$	$\frac{1}{3}\cdot\alpha_\ell+\frac{2}{3}\cdot\Delta$	$\frac{1}{15}\left[\Delta+4\alpha_\ell+\sqrt{\alpha_\ell^2+8\alpha_\ell\Delta+\Delta^2}\right]$
Π_i	$\frac{1}{9}\cdot\alpha_\ell^2$	$\left[\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta\right]^2$	$\frac{\left[\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta\right]^3}{\frac{1}{3}\cdot\alpha_\ell+\frac{2}{3}\cdot\Delta}$	$\frac{6}{15^3}\cdot\frac{\left[4\alpha_\ell+\Delta+\sqrt{\alpha_\ell+8\alpha_\ell\Delta+\Delta^2}\right]^3}{\alpha_\ell+\Delta+\sqrt{\alpha_\ell^2+8\alpha_\ell\Delta+\Delta^2}}$
Π_j	$\frac{1}{9}\cdot\alpha_\ell^2$	$\frac{\left[\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta\right]^3}{\frac{1}{3}\cdot\alpha_\ell+\frac{2}{3}\cdot\Delta}$	$\left[\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta\right]^2$	$\frac{6}{15^3}\cdot\frac{\left[4\alpha_\ell+\Delta+\sqrt{\alpha_\ell+8\alpha_\ell\Delta+\Delta^2}\right]^3}{\alpha_\ell+\Delta+\sqrt{\alpha_\ell^2+8\alpha_\ell\Delta+\Delta^2}}$

The three equilibria, case a, b and c correspond to points a, b and c in figure 2. Case d is not an equilibrium but included for completeness.

One can readily show the following lemma:

Lemma 2.:

$$4\alpha_\ell^2-6\alpha_\ell\Delta-\Delta^2 \geq 0 \implies \Pi_i\Big|_b > \Pi_i\Big|_a > \Pi_i\Big|_c$$

$$4\alpha_\ell^2-6\alpha_\ell\Delta-\Delta^2 \leq 0 \implies \Pi_i\Big|_b > \Pi_i\Big|_c > \Pi_i\Big|_a$$

Proof.:

Compare the relevant entries in table 1. □

Let the range Δ be a proxy for the variance of α, ($\mathrm{Var}(\alpha)= \frac{1}{12}\cdot\Delta^2$. Then if the variance is sufficinetly large relative to a_ℓ, both firms would prefer tthe asymmetric outcome over the symmetric outcome. As in either of the two cases, both firms prefer to be the igh capacity firm, there is no good basis for selecting a particular equilibrium as the most likely outcome.

The appealing feature of the symmetric equilibrium is that ex ante identical firms producing a homogeneous good get the same payoff in equilibrium. On the other hand it leads to some degree of indeterminacy as to who sets the high price in the final stage.

In the asymmetric equilibria, on the other hand, there is no doubth as to who sets the high price. Also note that the average profit in the asymmetric equilibria exceeds that in the symmetric:

$$\frac{1}{9}\cdot\alpha_\ell^2 < \frac{1}{2}\cdot\left[\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta\right]^2 + \frac{1}{2}\cdot\frac{\left[\frac{1}{3}\cdot\alpha_\ell+\frac{1}{6}\cdot\Delta\right]^3}{\frac{1}{3}\cdot\alpha_\ell+\frac{2}{3}\cdot\Delta}$$

$$\Longrightarrow$$

$$\frac{1}{108}\cdot\alpha_\ell^2\Delta + \frac{1}{18}\cdot\Delta^2\alpha_\ell + \frac{5}{432}\cdot\Delta^3 > 0$$

Hence if we allowed side payments, one of the asymmetric outcomes could be implemented as an equilibrium.

It should, though, be pointed out that the model rests on strong and at times special assumptions. Hence the results should be interpreted with care.

4. Asymmetric information.

In this section we consider a simple extension of our model to the case of asymmetric information. This case could signify one in which the worse informed is either an entrant or has been in the industry for a shorter period than the other. Without loss of generality, assume that firm 1 has access to the better information.

As noted in section 3.3, it is difficult to solve the capacity stage because the first-order conditions are non-linear. When considering asymmetric information these problems are exacerbated for two reasons. Firstly, it increases the number of non-linear equations which must be solved simultaneously making an analytical solution hard to obtain. This leaves one with the choice of either cooking up some very special examples as in section 4.1 below, or use numerical methods as in section 4.2 and 4.3 below. The second problem which arise is that when firm 1 has the better information, this information will be reflected in its capacity choice. Thus k_1 may fully or partially reveal the information of firm 1 to firm 2. Firm 1 then has an incentive to either hide its information, or to try and influence the inference drawn by firm 2 by biasing its choice of k_1. Such models of strategic information transmission are hard to solve, involving at best first order differential equations. For a solution of a simple leader follower model with private information, see Gal-Or(1987). In view of the difficultien of solving the model in the first place, we shall in the following assume that firm 1 does not attempt to bias its capacity choice and hence that k_1 fully reveal the information of firm 1.

In section 4.1 we consider a very special case which has an analytical solution. In section 4.2 and 4.3 we consider some numerical examples.

4.1 an analytical example.

We first work through an example with an analytical solution. Assume that firm 1 will be informed whether α is in the upper two-thirds or lower third of its support, i.e. $\alpha \in [\alpha_\ell, \hat{\alpha}]$ or $\alpha \in [\hat{\alpha}, \alpha_u]$, where $\hat{\alpha} = \alpha_\ell + \frac{1}{3} \cdot \Delta$. Firm 2 knows $\alpha \in [\alpha_\ell, \alpha_u]$. This information structure is common knowledge.

Firm 2 knows that firm 1 will have the better information and thus knows that its capacity in some cases will be too low, in some cases too high. Specifically it realises that if:

$$\begin{array}{lll} \alpha \in [\alpha_\ell, \hat{\alpha}] & \Rightarrow & k_2 > k_1 \\ \alpha \in [\hat{\alpha}, \alpha_u] & \Rightarrow & k_2 < k_1 \end{array}$$

Further in the second stage (because we assumed that k_1 fully revealed the information of firm 1), firm 2 will be able to infer the information of firm 1. Thus stage 2 is solved as in section 3.1. Given this firm 2 maximises the following in the first stage:

$$\max_{k_2} E(\Pi_2) = \max_{k_2} \left[\frac{1}{3} \cdot E(\Pi_2^h(k_2, k_1^\ell)) + \frac{2}{3} \cdot E(\Pi_2^\ell(k_2, k_1^h)) \right] \qquad (22)$$

Where k_1^ℓ and k_1^h is the capacity of firm 1 when α is low and high respectively and $E(\Pi_2^h(\cdot))$ and $E(\Pi_2^\ell(\cdot))$ are given by (10) and (15) respectively. Firm 1 maximises either:

$$\max_{k_1^\ell} E(\Pi_1) = E(\Pi_1^\ell) \quad \text{if} \quad \alpha \in [\alpha_\ell, \hat{\alpha}] \tag{23}$$

$$\max_{k_1^h} E(\Pi_1) = E(\Pi_1^h) \quad \text{if} \quad \alpha \in [\hat{\alpha}, \alpha_u] \tag{24}$$

The first order conditions are:

$$\frac{1}{3} \cdot \left. \frac{\partial E(\Pi_2^h(k_2, k_1^\ell))}{\partial k_2} \right|_{\alpha \in [\alpha_\ell, \hat{\alpha}]} + \frac{2}{3} \cdot \left. \frac{\partial E(\Pi_2^\ell(k_2, k_1^h))}{\partial k_2} \right|_{\alpha \in [\hat{\alpha}, \alpha_u]} = 0 \tag{25}$$

$$\left. \frac{\partial E(\Pi_1^\ell(k_1^\ell, k_2))}{\partial k_1^\ell} \right|_{\alpha \in [\alpha_\ell, \hat{\alpha}]} = 0 \tag{26}$$

$$\left. \frac{\partial E(\Pi_1^h(k_1^h, k_2))}{\partial k_1^h} \right|_{\alpha \in [\hat{\alpha}, \alpha_u]} = 0 \tag{27}$$

One possible solution is one in which both terms in (25) are zero. Denote the two terms (25a) and (25b) respectively. This case is particularly simple because we can solve (25a) and (26) seperately form (25b) and (27) so long as we ensure that the same k_2 solves both sets of equations. We can find the equilibrium capacities and profits by using column b and c in table 1 with α_ℓ and Δ suitably redefined[4)]. The capacities and ex ante expected profits are:

$$k_1 = \begin{cases} \frac{1}{3}\cdot\alpha_\ell + \frac{1}{18}\cdot\Delta & \text{for} \quad \alpha \in [\alpha_\ell, \hat{\alpha}] \\ \frac{1}{3}\cdot\alpha_\ell + \frac{5}{9}\cdot\Delta & \text{for} \quad \alpha \in [\hat{\alpha}, \alpha_u] \end{cases}$$

$$k_2 = \frac{1}{3}\cdot\alpha_\ell + \frac{2}{9}\cdot\Delta \qquad \text{for} \quad \alpha \in [\alpha_\ell, \alpha_u]$$

where $\Delta = \alpha_u - \alpha_\ell$.

$$E(\Pi_1^a) = \frac{1}{3}\cdot\frac{\left[\frac{1}{3}\cdot\alpha_\ell + \frac{1}{18}\cdot\Delta\right]^3}{\left[\frac{1}{3}\cdot\alpha_\ell + \frac{2}{9}\cdot\Delta\right]} + \frac{2}{3}\cdot\left[\frac{1}{3}\cdot\alpha_\ell + \frac{2}{9}\cdot\Delta\right]^2 \tag{28}$$

$$E(\Pi_2^a) = \frac{1}{3}\cdot\left[\frac{1}{3}\cdot\alpha_\ell + \frac{1}{18}\cdot\Delta\right]^2 + \frac{2}{3}\cdot\frac{\left[\frac{1}{3}\cdot\alpha_\ell + \frac{2}{9}\cdot\Delta\right]^3}{\left[\frac{1}{3}\cdot\alpha_\ell + \frac{5}{9}\cdot\Delta\right]} \tag{29}$$

If both firms had access to the better information we are back in the case considered in section 3, where payoffs are given in table 1. Depending on which of the three possible equilibria are expected to prevail, we get the following ex ante expected profits.[5)]

The symmetric case (s):

$$E(\Pi_s^I) = \frac{1}{3}\cdot\frac{1}{9}\cdot\alpha_\ell^2 + \frac{1}{3}\cdot\left[\frac{1}{3}\cdot\alpha_\ell + \frac{1}{9}\cdot\Delta\right]^2$$

The asymmetric case where firm h is the high capacity firm, firm ℓ the low capacity firm:

$$E(\Pi_h^I) = \frac{1}{3}\cdot\left[\frac{1}{3}\cdot\alpha_\ell + \frac{1}{18}\cdot\Delta\right]^2 + \frac{2}{3}\cdot\left[\frac{1}{3}\cdot\alpha_\ell + \frac{2}{9}\cdot\Delta\right]^2$$

$$E(\Pi_\ell^I) = \frac{1}{3}\cdot\frac{\left[\frac{1}{3}\cdot\alpha_\ell + \frac{1}{18}\cdot\Delta\right]^3}{\left[\frac{1}{3}\cdot\alpha_\ell + \frac{2}{9}\cdot\Delta\right]} + \frac{2}{3}\cdot\frac{\left[\frac{1}{3}\cdot\alpha_\ell + \frac{2}{9}\cdot\Delta\right]^3}{\left[\frac{1}{3}\cdot\alpha_\ell + \frac{5}{9}\cdot\Delta\right]}$$

Finally, if neither had any information we get

The symmetric case:

$$E(\Pi_s^u) = \frac{1}{9}\cdot\alpha_\ell^2$$

$$E(\Pi_h^u) = \left[\frac{1}{3}\cdot\alpha_\ell + \frac{1}{6}\cdot\Delta\right]^2$$

$$E(\Pi_\ell^u) = \frac{\left[\frac{1}{3}\cdot\alpha_\ell + \frac{1}{6}\cdot\Delta\right]^3}{\left[\frac{1}{3}\cdot\alpha_\ell + \frac{2}{3}\cdot\Delta\right]}$$

Comparing the different profit levels, we can show the following ranking:

The symmetric case:

$$E(\Pi_1^a) > E(\Pi_s^I) > E(\Pi_2^a) > E(\Pi_s^u)$$

The asymmatric case:

(i) $E(\Pi_h^I) > E(\Pi_h^a) > E(\Pi_h^u)$

(ii) $E(\Pi_\ell^a) > E(\Pi_\ell^I) > E(\Pi_\ell^u)$

(iii) $E(\Pi_h^I) > E(\Pi_h^a) > E(\Pi_\ell^a) > E(\Pi_\ell^I)$

In both the symmetric and the asymmetric case, information has a value for both firms, i.e. $E(\Pi^a) > E(\Pi^u)$. Whether or not the informed has an incentive to pass on his information or the uninformed want to obtain it depends on which of the three equilibria will arise in the case where both have the same information. In the symmetric case the informed do not want to disseminate information whereas the uninformed want to obtain it. In the asymmetric case where the best informed also become the high capacity setter following the information dissemination, the informed is willing to share his information, even if restricted to truth-telling. The uninformed on the other hand does not want this information (see (iii) above). In the asymmetric case where the best informed become the low capacity firm following information dissemination, this is reversed.

4.2 Numerical examples.

We want to get an idea of what happens if we change the information set of firm 1 by partitioning the set $[\alpha_\ell, \alpha_u]$. Let n be the number of partitions of $[\alpha_\ell, \alpha_u]$ of equal length. Thus contrary to section 4.1 above we consider symmetric partitions which in itself may change the previous results, i.e the results may depend on the partitions used. The numerical results are given in table 2 below. The capacities and profits are averages.

Table 2.

α_ℓ	α_u	n	Cour-not output	No Sharing k_1	No Sharing k_2	Sharing k_i	No Sharing Π_1	No Sharing Π_2	Sharing Π_i
30	31	1	10.2	10.500	10.500	10.50	100.00	100.00	100.0
		2		10.300	10.291	10.33	102.13	102.12	101.7
		3		10.259	10.259	10.28	102.45	102.43	102.2
		4		10.229	10.229	10.25	102.75	102.73	102.5
		5		10.220	10.220	10.23	102.84	102.83	102.7
		6		10.208	10.209	10.22	102.96	102.94	102.8
		7		10.204	10.204	10.21	103.00	102.98	102.9
30	33	1	10.5	11.500	11.500	11.50	100.0	100.0	100.0
		2		10.873	10.867	11.00	106.7	106.5	105.1
		3		10.780	10.779	10.83	107.6	107.4	106.5
		4		10.688	10.690	10.75	108.5	108.3	107.7
		5		10.661	10.663	10.70	108.8	108.6	108.2
		6		10.626	10.628	10.66	109.1	109.0	108.6
		7		10.613	10.615	10.64	109.3	109.1	108.8
30	36	1	11.0	13.000	13.000	13.00	100.0	100.0	100.0
		2		11.740	11.720	12.00	114.2	113.6	110.5
		3		11.562	11.560	11.66	115.9	115.3	114.1
		4		11.376	11.380	11.50	117.8	117.2	115.8
		5		11.323	11.333	11.40	118.3	117.7	117.0
		6		11.251	11.260	11.33	119.0	118.4	117.7
		7		11.227	11.237	11.28	119.2	118.6	118.2

Column 7 and 10 gives the symmetric solution when both firms have access to the better information. 6) The most remarkable result of our numerical examples is that neither firm prefer to share the information of firm 1. In these examples this is caused by the fact that due to uncertainty the worse informed (firm 2) choose an average capacity which is lower than under information sharing. This allows the better informed (firm 1) also to choose a lower capacity, leading to lower total output and higher profit for both firms.

Secondly from the tabel it seems as if output and profit converge to the Cournot level as firm 1 moves towards perfect information. We can easily show that this is the case when both firms share the information. Let n be the number of partitions of $[\alpha_\ell, \alpha_u]$ of equal length. Then as $n \to \infty$ firms become perfectly informed about α. Define $\delta \equiv \Delta/n$, and let Π_n be the profit of a firm when the number of partitions is n. For the three equilibria, this can be written as

Case a:

$$\Pi_n^a = \frac{1}{9}\cdot\frac{1}{n}\cdot\left[(\alpha_\ell)^2 + (\alpha_\ell+\delta)^2 + \cdots + (\alpha_\ell+(n-1)\cdot\delta)^2\right]$$

$$= \frac{1}{9}\cdot\frac{1}{n}\cdot\sum_{i=0}^{n-1}(\alpha_\ell+i\cdot\delta)^2 = \frac{1}{9}\cdot\left[\alpha_\ell^2 + \frac{n-1}{n}\alpha_\ell\Delta + \frac{(n-1)(2\cdot n-1)}{6\cdot n^2}\cdot\Delta^2\right]$$

$$\lim_{n\to\infty}\Pi_n^a = \frac{1}{9}\cdot\left[\alpha_\ell^2 + \alpha_\ell\cdot\Delta + \frac{1}{3}\cdot\Delta^2\right]$$

Case b:

$$\Pi_n^b = \frac{1}{9}\cdot\frac{1}{n}\cdot\sum_{i=0}^{n-1}(\alpha_\ell+i\cdot\delta+\frac{1}{2}\cdot\delta)^2$$

$$= \frac{1}{9}\cdot\left[(\alpha_\ell+\frac{1}{6}\cdot\frac{\Delta}{n})^2 + \frac{n-1}{n}\cdot\Delta\cdot(\alpha_\ell+\frac{1}{6}\cdot\frac{\Delta}{n}) + \frac{(n-1)(2\cdot n-1)}{6\cdot n^2}\cdot\Delta^2\right]$$

$$\lim_{n\to\infty}\Pi_n^b = \frac{1}{9}\cdot\left[\alpha_\ell^2 + \alpha_\ell\cdot\Delta + \frac{1}{3}\cdot\Delta^2\right]$$

Case c:

$$\Pi_n^c = \frac{1}{9}\cdot\frac{1}{n}\cdot\sum_{i=0}^{n-1}\frac{(\alpha_\ell+i\cdot\delta+\frac{1}{2}\cdot\delta)^2}{(\alpha_\ell+i\cdot\delta+2\cdot\delta)}$$

$$= \frac{1}{9}\cdot\frac{1}{n}\cdot\sum_{i=0}^{n-1}\left[(\alpha_\ell+i\delta+2\delta)^2 - \frac{9}{2}\cdot(\alpha_\ell+i\delta+2\delta) + \frac{27}{2}\delta^2 - \frac{\frac{27}{8}\cdot\delta^3}{(\alpha_\ell+i\delta+2\delta)}\right]$$

$$= \frac{1}{9}\cdot\left[(\alpha_\ell+2\cdot\frac{\Delta}{n})^2 + \frac{n-1}{n}\cdot(\alpha_\ell+2\cdot\frac{\Delta}{n})\cdot\Delta + \frac{(n-1)(2\cdot n-1)}{6\cdot n^2}\cdot\Delta^2\right]$$

$$- \frac{\Delta}{2n}\cdot\left[(\alpha_\ell+2\cdot\frac{\Delta}{n}) + \frac{n-1}{n}\cdot\Delta + \frac{3}{2}\cdot\frac{\Delta^2}{n^2}\right] - \frac{1}{9}\cdot\frac{1}{n}\cdot\sum_{i=0}^{n-1}\frac{27\cdot\delta^3}{(\alpha_\ell+i\delta+2\delta)} \qquad (30)$$

Note that for i=0,1,... $0 \leq \frac{27\cdot\delta^3}{(\alpha_\ell+i\delta+2\delta)} \leq \frac{\delta^3}{\alpha_\ell+2\cdot\delta}$ and

$\lim_{n\to\infty} \frac{1}{n}\cdot\sum_{i=0}^{n-1}\frac{\left[\frac{\Delta}{n}\right]^3}{\alpha_\ell+2\cdot\frac{\Delta}{n}} = 0$. Hence the linit of the last term in (30) is zero, and we find:

$$\lim_{n\to\infty} \Pi_n^c = \frac{1}{9}\cdot\left[\alpha_\ell^2 + \alpha_\ell\cdot\Delta + \frac{1}{3}\cdot\Delta^2\right]$$

If both firms had perfect information prior to choosing capacity, expected profit of firm i would be:

$$E(\Pi) = \frac{1}{\Delta}\cdot\int_{\alpha_\ell}^{\alpha_u}\frac{\alpha^2}{9}\cdot d\alpha = \frac{1}{9}\cdot\frac{\alpha_u^2+\alpha_u\cdot\alpha_\ell+\alpha_\ell^2}{3} = \frac{1}{9}\cdot\left[\alpha_\ell^2 + \alpha_\ell\cdot\Delta + \frac{1}{3}\cdot\Delta^2\right]$$

$$= \lim_{n\to\infty} \Pi_n^a = \lim_{n\to\infty} \Pi_n^b = \lim_{n\to\infty} \Pi_n^c$$

So all three equilibria converge to the perfect information equilibrium.

It is difficult to show convergence for the case of asymmetric information. It involves finding what the solution to a system of n+1 non-linear equations converges to as $n \to \infty$. We can look at the limit at which one firm, firm 1, has perfect information the other, firm 2, no information. In this case the reaction function of the informed become

$$k_1(\alpha) = \frac{\alpha - k_2}{2} \tag{25}$$

The first order condition of the uninformed can be written as:

$$\frac{1}{\Lambda} \int_{\alpha_\ell}^{\alpha_u} (k_2 - \frac{\alpha - k_1(\alpha)}{2}) \cdot d\alpha = 0 \tag{26}$$

The solution to (25) and (26) is

$$k_2 = \frac{E(a)}{3} \qquad k_1 = \frac{\alpha}{2} - \frac{1}{6} \cdot E(\alpha)$$

The equilibrium price and profits are:[7)]

$$P = \frac{\alpha}{2} - \frac{1}{6} \cdot E(\alpha); \quad E(\Pi_1) = \frac{1}{9} \cdot E(\alpha)^2 + \frac{1}{4} \cdot \sigma_\alpha^2; \quad E(\Pi_2) = \frac{1}{9} \cdot E(\alpha)^2$$

If both had access to full information profits would be:

$$E(\Pi_i) = \frac{1}{9} \cdot E(\alpha)^2 + \frac{1}{9} \cdot \sigma_\alpha^2 \qquad i=1,2$$

So in the limit, firm 2 value the information of firm 1, but firm 1 prefers not to share its information with firm 2.

4.3 Another example.

We consider an example where both firms have access to private information. Assume that firm 1 knows whether α is in the lower third or the upper two-thirds and that firm 2 knows whether α is in the lower two-thirds or the upper third. We can illustrate this in a diagram:

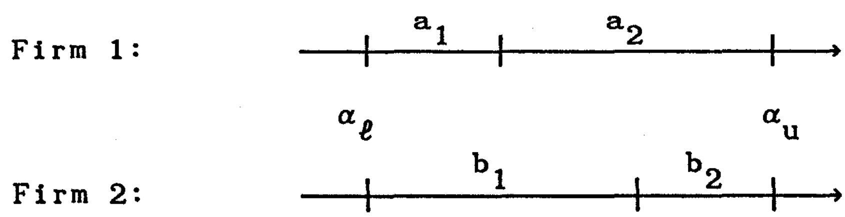

figure 3.

Now there are three possible cases: $\alpha \in a_1 = [\alpha_\ell, \alpha_1]$, $\alpha \in b_1 \cap a_2 = [\alpha_1, \alpha_2]$, and $\alpha \in b_2 = [\alpha_2, \alpha_u]$.

Case 1: Firm 1 knows that firm 2 has observed $\alpha \in b_1$. Hence it knows that it will get the lower capacity regardless and hence maximise (15). Firm 2 believes with probability 1/2 that firm 1 has observed $\alpha \in a_1$ and with probability 1/2 $\alpha \in a_2$. In the former case it will have the higher capacity and maximises (10) in the latter it will have the lower capacity and maximises (15). Thus firm 2's problem is:

$$\max_{k_2} \left[\frac{1}{2} \cdot \Pi_2^h \Big|_{\alpha \in [\alpha_\ell, \alpha_1]} + \frac{1}{2} \cdot \Pi_2^\ell \Big|_{\alpha \in [\alpha_1, \alpha_2]} \right]$$

Case 2: Firm 1 believes with probability 1/2 that firm 2 has observed $\alpha \in b_1$ with probability 1/2 $\alpha \in b_2$. Firm 2 believes with probability 1/2 that firm 1 has observed $\alpha \in a_1$ with probability 1/2 $\alpha \in a_2$. The respective problems are then:

$$\max_{k_1} \left[\frac{1}{2}\cdot \Pi_1^h \Big|_{\alpha\in[\alpha_1,\alpha_2]} + \frac{1}{2}\cdot \Pi_1^\ell \Big|_{\alpha\in[\alpha_2,\alpha_u]} \right]$$

$$\max_{k_2} \left[\frac{1}{2}\cdot \Pi_2^h \Big|_{\alpha\in[\alpha_\ell,\alpha_1]} + \frac{1}{2}\cdot \Pi_2^\ell \Big|_{\alpha\in[\alpha_1,\alpha_2]} \right]$$

Case 3: Firm 2 knows that firm 1 has observed $\alpha \in a_2$. Firm 1 believes with probability 1/2 that firm 2 has observed $\alpha \in b_1$ with probability 1/2 $\alpha \in b_2$. The problem of firm 1 is then:

$$\max_{k_1} \left[\frac{1}{2}\cdot \Pi_1^h \Big|_{\alpha\in[\alpha_1,\alpha_2]} + \frac{1}{2}\cdot \Pi_1^\ell \Big|_{\alpha\in[\alpha_2,\alpha_u]} \right]$$

We get the following numerical results, shown in table 3 below, where k_1 and k_2 is capacity of firm 1 and 2 respectively when neither firm share the information, and k_i is the capacity of firm i when both firm share their information.

Table 3.

(α_ℓ, α_u)	cap	case 1 $\alpha \in [\alpha_\ell, \alpha_1]$	case 2 $\alpha \in [\alpha_1, \alpha_2]$	case 3 $\alpha \in [\alpha_2, \alpha_u]$	A priori expected
(30,36)	k_1	10.27	12.37	11.38	11.34
	k_2	11.51	10.59	12.81	11.64
	k_i	11.00	11.67	12.33	11.67
(30,33)	k_1	10.12	11.21	10.68	10.69
	k_2	10.78	10.27	11.41	10.82
	k_i	10.50	10.83	11.17	10.83
(33,36)	k_1	11.11	12.21	11.68	11.67
	k_2	11.79	11.27	12.41	11.83
	k_i	11.50	11.83	12.17	11.83

So firm 2 has higher a priori expected capacity, but capacity below the information-sharing capacity. Both firms are above expected Cournot capacity.

Corresponding profits are shown in table 4 below, where Π_i is the profit of firm i if both shared the information.

Table 4.

(α_ℓ,α_u)	cap	case 1 $\alpha\in[\alpha_\ell,\alpha_1]$	case 2 $\alpha\in[\alpha_1,\alpha_2]$	case 3 $\alpha\in[\alpha_2,\alpha_u]$	A priori expected
(30,36)	Π_1	95.87	125.55	123.89	115.10
	Π_2	107.46	107.48	139.52	118.15
	Π_i	100.00	113.78	128.44	114.07
(30,33)	Π_1	97.43	112.62	111.40	107.15
	Π_2	103.87	103.21	119.05	108.71
	Π_i	100.00	106.78	113.78	106.85
(33,36)	Π_1	118.11	134.87	133.49	128.82
	Π_2	125.29	124.47	141.88	130.55
	Π_i	121.00	128.44	136.11	128.52

From the numerical examples, there are no signs of firms wanting to share information.

Our examples suggest some tentative conclusions. Firstly, when information is imparted involuntarily through plans or acts, there seems little scope for voluntary information transmission. In the examples, firms choose their capacity in greater ignorance than when they choose prices. It seems to be the case that the added uncertainty implies a bias towards lower capacity, which helps to keep prices high. The reduced uncertainty in the pricing stage helps firms choose a "better" price. Thus firms in a sense prefer greater uncertainty in the first stage coupled with less uncertainty in the second stage.

Secondly, information need not have a positive value for the worse informed. This result is caused by ignorance in the first stage being potentially profitable because it biases total capacity downwards. This implies that there need not be anything which drives a firm to obtain more information prior to the planning stage.

Again caution is necessary in interpreting the results of the examples. They do not necessarily generalise, and it seems very likely that the results hinges on the functional forms choosen, as well as the information partitions considered.

5. concluding remarks.

In this paper we have shown that under uncertainty, firms will not set the same price and hence the analogy with the Cournot outcome no longer hold. The high capacity firm is shown to set the high price and earn the higher profit. In the capacity stage 3 equilibria was found, one symmetric, and two asymmetric (differing only in the labelling of firms). Thus the subgame perfect equilibrium may involve ex ante identical firms producing a homogeneous good having both different capacities and different prices.Further it is shown that firms install larger capacity under uncertainty that they would have done had the mean of the random variable been known to occur with certainty.

Secondly private asymmetric information was considered. We showed that there did not seem any incentives for information sharing between firms. Further numerical examples indicate that there may be cases where both firms would prefer to be ignorant when choosing capacity and informed when choosing prices. This question is left for future research. For future research it would also be of interest to consider other information structures.

Another result which should be noted is that market share measured as a fraction of total industry output supplied is a random variable, because the sales of the high price firm is random whereas the sales of the low price firm is deterministic. This also implies that the two market shares need not follow the same distribution. Now this problem is likely to generalise to a n-firm model, where some firms will always be at their capacity limit whereas others will act as buffers. This has consequences for the choice of estimation technique if market share is used as an explanatory variable in an econometric model, because not only would the explanatory variable be stochastic, but the error term need not be identically distributed. This cast severe doubt on e.g. ordinary least squares as the best estimator, and gives further reasons for extensive testing of the assumptions underlying the estimation technique.

Footnotes:

1) An alternative rule often called reservation price rationing originates from Edgeworth(1925). Independent of whether consumers are homogeneous or heterogeneous, a random selection is allowed to purchase their entire demand. This is a kind of first come first served rule where the place in the queue is allotted randomly. Note is that parallel rationing is much simpler to work with because the residual demand of firm i depends on the price of firm j in a very trivial manner, as it only affects the point of discontinuity of the demand function.

2) This equilibrium concept is related to the one used in Eaton and Kierzkowski(1984) who assume:

"Each firm takes the other firm's price as given when contemplating price *reductions*. In considering price increases, however, each firm takes into account the incentives it may create for the other form to *lower* its price. In equilibrium each firm will charge the highest price it can without provoking a price cut by the other firm."(pp. 102, their italics).

3) Although Stackelberg(1934) (not surpricingly) did not use game theoretic arguments, he was well aware of the difficulties in attaining an equilibrium where en endogeneous leader exist. For a much later exposition in English, see Gal-Or(1986) or Dowrick(1986). It is worth mentioning that Stackelberg(1934) has not been translated into english. The book by Stackelberg which has been translated into english, Stackelberg(1952), is a textbook containing only a brief outline of the analysis of duopoly carried out in Stackelberg(1934) and not Stackelberg(1934) as some seems to believe.

4) Solving (25a) and (26) use $\alpha_\ell=\alpha_\ell$ and $\Lambda=\Lambda/3$. Solving (25b) and (27) use $\alpha_\ell=\alpha_\ell+\Lambda/3$ and $\Lambda=2\cdot\Lambda/3$.

5) The profits are found as follows. For $\alpha\in[\alpha_\ell,\hat{\alpha}]$ set $\alpha_\ell=\alpha_\ell$ and $\Lambda=\Lambda/3$ in table 1. For $\alpha\in[\hat{\alpha},\alpha_u]$ set $\alpha_\ell=\alpha_\ell+\Lambda/3$ and $\Lambda=2\cdot\Lambda/3$ in table 1. Finally weight the two expressions by 1/3 and 2/3 respectively.

6) If the informed firm was certain that the outcome would be the asymmetric outcome in which it has the high capacity, it would prefer to submit its information to the uninformed even it had to do so truthfully. On the other hand, if it had the lower capacity, no such incentives exist for the informed firm.

7) Where $E(\alpha)=(\alpha_u+\alpha_\ell)/2$ and $Var(\alpha)=(\alpha_u-\alpha_\ell)^2/12$.

Appendix V.A.

From (15) we know that given $k_1 > k_2$, $\Pi_1^h > \Pi_2^\ell$ and $\Pi_1^\ell > \Pi_2^h$. With linear capacity costs this still holds because:

$$\Pi_1^{hp} - \Pi_2^{lp} = \frac{k_1-k_2}{k_1}(\Pi_1^{hp} - bk_1) > 0$$

$$\Pi_2^{hp} - \Pi_1^{lp} = \frac{k_2-k_1}{k_2}(\Pi_2^{hp} - bk_2) < 0$$

Using (15) we can write:

$$\Pi_1^h - \Pi_1^\ell = \Pi_1^h - \frac{k_1}{k_2}\cdot\Pi_2^h = -\frac{k_1}{k_2}\cdot(\Pi_2^h - \frac{k_2}{k_1}\cdot\Pi_1^h) = -\frac{k_1}{k_2}\cdot(\Pi_2^h - \Pi_2^\ell) \quad \text{(A1)}$$

Note that (A1) is independent of capacity costs.

Define $\Lambda \equiv \alpha_u - \alpha_\ell$. Now

$$\Pi_i^h - \Pi_i^\ell = \frac{1}{27\cdot\Lambda}\cdot\left[(\varphi(k_i))^3 - (\alpha_\ell-k)^3 - \frac{k_i}{k_j}\cdot\left[(\varphi(k_j))^3 - (\alpha_\ell-k)^3\right] \right]$$

$$\equiv \frac{1}{27\cdot\Lambda}\cdot\Gamma(\Lambda,k_i,k_j,\cdot)$$

At $\Lambda=0$, we have $\Gamma = 0$. Also:

$$\frac{\partial\Gamma}{\partial\Lambda} = \frac{3}{2}\cdot\varphi(k_i)\cdot k_i - \frac{k_i}{k_j}\cdot\frac{2}{2}\cdot\varphi(k_j)\cdot k_j \geq 0 \qquad \text{for} \quad \Lambda \geq 0$$

$$\frac{\partial(\Pi_i^h-\Pi_i^\ell)}{\partial\Lambda} = -\frac{1}{27\cdot\Lambda^2}\cdot\Gamma(\cdot) + \frac{1}{27\cdot\Lambda}\cdot\frac{\partial\Gamma}{\partial\Lambda} \quad \text{(A2)}$$

Also

$$\Pi_i^h - \Pi_i^\ell \longrightarrow \frac{(\alpha_\ell - k)\cdot(k_i - k_j)}{18} \qquad \text{for } \Lambda \longrightarrow 0$$

Hence Γ must be positive for $\Lambda \geq 0$, because if Γ was negative then (A2) would surely be positive, but then profits is an increasing function of Λ over its range and positive at $\Lambda=0$, a contradiction.

This implies that $\Pi_i^h > \Pi_i^\ell$ and from (A1) we have $\Pi_j^h < \Pi_j^\ell$. Finally

$$\Pi_i^\ell > \Pi_j^\ell \qquad \Rightarrow$$

$$\frac{2}{3}(\alpha_\ell - k)(k_i - k_j) - \frac{(\alpha_\ell - k)^3}{27\cdot\Lambda}\left[\frac{k_i}{k_j} - \frac{k_j}{k_i}\right] + \frac{1}{27\Lambda}\left[\frac{k_i}{k_j}(\varphi(k_i))^3 - \frac{k_j}{k_i}(\varphi(k_j))^3\right] \geq 0$$

But from above we know that $\Gamma > 0$. Then the inequality follows. Finally $k_i = k_j$ follows from (15). Q.E.D.

CHAPTER VI

CONSISTENT CONJECTURES WITH INFORMATION TRANSMISSION*

by

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1. INTRODUCTION

The use of conjectural variations in the analysis of quantity-setting oligopoly is widespread. The reason is that differing conjectures permit a continuum of equilibria, ranging from an apparent Bertrand competitive equilibrium, general negative conjectural equilibria and then through Cournot zero conjectures to positive conjectures and apparent collusion. However, two questions of general relevance to oligopoly theory are particularly appropriate to conjectural variations models. First, what is the benefit of being able to catalogue a large array of different equilibria relating to differing conjectural variations if there is no way of selecting the particular equilibrium that is appropriate in a specific case? Is it not possible to restrict the set of possible conjectures? Although some progress has been made in making the choice of game strategy endogenous in sequential-game models (see Kalai and Stanford, 1985), there appears that little has been achieved to date in terms of a one-stage game.[1/]

The second and related question concerns the rationality or consistency of conjectural variations. A number of papers beginning with Bresnahan (1981)[2/] have argued that only consistent conjectures are valid. Thus an equilibrium, based on a firm's belief that competitors will increase their outputs in response to an increase in its output, cannot be sustained if a different response (for instance a reduction in competitors' outputs) would occur if the firm experimented by increasing its output from the supposed equilibrium position. If all inconsistent conjectural equilibria are dismissed as invalid, then in the simplest (linear demand, homogeneous product, constant marginal cost) case, only competitive equilibria are consistent (e.g. Perry, 1982, Proposition 1). If each firm expects that others will "make room" for any increased output it chooses to produce, then a competitive industry supply level will be achieved and price will remain equal to marginal cost, irrespective of an individual firm's changes in output.

One might reject the argument that consistency is a necessary or even desirable property for a conjectural variations equilibrium.[3/] Experimentation to test consistency may appear difficult to carry out without one firm's experiments interfering with those of another. Also, one might argue that the basis for the conjectures lies outside the simple model and that their justification is in terms of these exogenous considerations. It would appear that one has to make such arguments in order to allow other conjectures than the competitive conjecture to be of any interest. However, to dismiss the property of consistency as a requirement loses the very real possibility of determining conjectural variations endogenously and thus being able to focus on one or more equilibria of interest from the set of conjectural variations equilibria. A more intriguing prospect is to enlarge the conjectural variations model so as to allow any specific conjecture to be consistent in particular (but not general) circumstances. In this paper we will take this alternative route and outline a model where demand uncertainty permits a quantity strategy to be interpreted both in the usual terms of an output plan and additionally as a signal relating to demand conditions. By making this extension of the basic model it will be possible to restrict the number of consistent conjectures to just two. One will again be the competitive conjecture, but the other may be from anywhere in the complete range of possible conjectures depending only on the parameters relating to information transmission.

The general argument of the model can be put very simply. Suppose firm i announces an increase in its output. In a market with no uncertainty, other firms would simply see the remaining market as smaller and cut back their planned outputs. However, if firms are uncertain about the strength of market demand they will be heartened by firm i's apparent confidence in the market. They will revise their expectations in an optimistic direction, and this revision will have a counteracting effect on supply plans. Thus even a positive conjecture may be consistent if the increase in optimism is sufficiently large.

The following timing of events is envisaged.

(i) Firms obtain their private information and use this to decide on an output plan which is then communicated (voluntarily or involuntarily) to all other firms.

(ii) Since firms cannot hide their output plans from each other, a firm's plan gives an (imperfect) clue as to its own private market information to its competitors.

(iii) Each firm can now extract information about the level of demand from the output plans of others and can add this to its own private information.

(iv) In the light of all the firm's information, its output plan is revised and announced, given a conjecture concerning the reaction of its competitors to both the market preemption and market information aspects of the plan. This conjecture is consistent with the firm's own reaction function.

(v) Given their augmented information and conjectures, the revised output plans could be an industry equilibrium. That is individual firms may predict the outcome of the Nash game in outputs.

(vi) Plans are implemented only when all firms are in equilibrium, and actual outputs comply with firms' last announced plans. This latter assumption effectively prevents firms from pretending one plan while executing another.

(vii) Since the last plan announced by the firm will be carried out, only this plan has any actual effect on other firms' behaviour. There is no point in a firm attempting to mislead others by announcing an initial plan, the information content of which is to be contradicted by later plans. If the plan is revised in a way which changes the character of the information contained in it, other firms will forget the old information, and incorporate the new information in their market expectations and output plans.

The fact that firms output plans become full commitments to production is an important simplification.[4/] Without this property, firms would trade off the cost of deviating from such plans with the benefits of fake representation of information and intentions. Such behaviour is the subject of further research. However, it is interesting to consider the information transmission mechanism in a model when final plans are implemented in order to assess the impact of the quality of information transmission on equilibrium behaviour. It is still possible for firms to choose final output plans which understate market demand: the constraint on them is that they actually have to be bound by such plans. The full commitment to the final plan may arise due to the public nature of firms' contracts with input suppliers or to industrial espionage. It may also reflect plans as responses to surveys by manufacturers' associations; in this case firms may wish to retain future credibility by responding their true current intentions. Examples of such surveys are widespread although they mostly relate to trends in business confidence and other qualitative factors.[5/] Our analysis here yields the possibility of an interesting motivation for such indicative planning mechanisms: to increase the coordination of industry supply and thus the apparent collusion of suppliers.

In Section II, a simple model with quality-setting firms is augmented to incorporate cost and demand uncertainty and information transmission via output plans. The set of consistent conjectures is derived. Section III categorises the conjectural equilibria in a series of propositions. It is shown that there exist underlying parameters of the model such that any conjectural response between competition and full apparent collusion can be consistent. Some comparative static analysis is reported. It is also shown that a continuous adjustment process of output plans is stable so that the conjectural equilibrium would be achieved whatever initial output plans were assumed. Section IV summarises conclusions and reviews the assumptions of the analysis. Some suggestions for further research are incorporated.

II. CONSISTENT CONJECTURES IN AN EQUILIBRIUM WITH INFORMATION TRANSMISSION

We will only consider an n-firm symmetric case with a linear market demand for a homogeneous product:

$$p = \alpha' - Q \tag{1}$$

where p is product price and Q is market supply. The intercept α' is a random variable, normally distributed $N(\bar{\alpha}', \sigma_\alpha^2)$. Variable costs of production for the i^{th} firm are

$$C_i(q_i) = (\bar{m} + \eta_i)q_i$$

where q_i is the i^{th} firm's output and η_i is a random variable, normally distributed $N(Q, \sigma_\eta^2)$. The variance of η_i is invariant across firms in order that firms are ex ante identical. The η_i's are assumed uncorrelated across firms so that above expected costs in one firm does not imply above or below expected costs in any other. To simplify notation define $\alpha \equiv \alpha' - \bar{m}$. $E(\alpha) = \bar{\alpha}' - \bar{m} = \bar{\alpha}$, so that $\alpha \sim N(\bar{\alpha}, \sigma_\alpha^2)$.

We shall assume that each firm receives perfect information (a perfect signal) concerning its own cost (η_i) prior to choosing a strategy, but no direct information concerning the other firms' costs. Each firm also receives an imperfect signal S_i on the realisation of α prior to choosing a strategy. The signal is assumed of the form

$$S_i = \alpha + \varepsilon_i \tag{2}$$

where ε_i is normally distributed $N(0, \sigma_\varepsilon^2)$, and $E(\varepsilon_i, \varepsilon_j) = 0$ $\forall i,j \; i \neq j$. Further it is assumed that α, η_i and ε_i are uncorrelated.

Our model is one of differential information, i.e. no firm believes that any firm has superior or inferior information to any other. Given all this we can summarise what is assumed common knowledge for all firms.

$$\left.\begin{array}{ll}
\alpha \sim N(\bar{\alpha}, \sigma_\alpha^2) & \forall i \\
\eta_i \sim N(0, \sigma_\eta^2) & \forall i \\
\varepsilon_i \sim N(0, \sigma_\varepsilon^2) & \forall i \\
E(\alpha, \eta_i) = E(\alpha, \varepsilon_i) = 0 & \forall i \\
E(\varepsilon_i, \eta_j) = 0 & \forall i, j \\
E(\varepsilon_i, \varepsilon_j) = E(\eta_i, \eta_j) = 0 & \forall i, j \quad i \neq j
\end{array}\right\} \tag{3}$$

In the absence of any other information, firm i would use (2) to predict the realisation of α, $E(\alpha|S_i)$. Given the prior on α and the normality assumptions.[6/] $E(\alpha|S_i)$ is the convex combination of S_i and $\bar{\alpha}$:

$$E(\alpha|S_i) = t\, S_i + (1-t)\bar{\alpha} \tag{4}$$

where $$t = \sigma_\alpha^2/(\sigma_\alpha^2 + \sigma_\varepsilon^2) \tag{5}$$

It would then choose its output q_i to maximise expected profits, given η_i and S_i

$$\max_{q_i} E(\Pi_i|S_i, \eta_i) = \max_{q_i} [(t\, S_i + (1-t)\bar{\alpha} - \eta_i - Q).q_i] \tag{6}$$

Now consider that firm i has a complete conjecture concerning the rest of the industry's behaviour. Let this be of the form

$$Q_{-i} = a + bS'_{-i} + cq_i \tag{7}$$

where Q_{-i} is the output of all the other firms in the industry and S'_{-i} is the "aggregate" signals on which the other firms base their choice of strategies. The form of (7) reflects the fact that if firm i observes Q_{-i} it would be able to infer additional information on α.

Due to the structure of the model, the first-order condition for firm j must be linear in S_j and η_j. Hence Q_{-i} must be linear in all S_j, η_j, $j \neq i$. The signal on α obtained from Q_{-i} can thus be written as

$$\begin{aligned} S'_{-i} &= \frac{1}{n-1} \sum_{j\neq i} (S_j - k\eta_j) = \alpha + \frac{1}{n-1} \sum_{j\neq i} (\varepsilon_j - k\eta_j) \\ &= \frac{1}{n-1} \sum_{j\neq i} S'_j \qquad\qquad = \alpha + \theta_i \end{aligned} \tag{8}$$

where $S'_j = S_j - k\eta_j$, $\theta_i = (\sum_{j\neq i} (\varepsilon_j - k\eta_j))/(n - 1)$

and k is a parameter which reflects the relative importance of S_j and η_j on other firms' outputs. Observing Q_{-i} only partially reveals the information held on α by the other $(n-1)$ firms (the S_j) due to the "noise" arising from random cost (η_j); only the S'_j are inferred.

If firms have to announce their output plans prior to carrying them out, then Q_{-i} is available to the i^{th} firm. From its conjectural function (7), S'_{-i} can be inferred. Thus any candidate for an equilibrium set of output plans $q_1, q_2 \ldots, q_n$ must incorporate the information contained in $S'_{-i}, S'_{-2}, \ldots, S'_{-n}$ respectively. Each firm i has two signals, S_i and S'_{-i} to use for predicting the demand parameter α.

Firm i's information on α given in (2) and (8) can be summarised, see Jaffe and Winkler (1976, p 50, 57-58), as

$$\tilde{S}_i = \delta S_i + (1-\delta)\, S'_{-i} \tag{9}$$

$$\text{VAR}\,(\tilde{S}_i) = \sigma_\alpha^2 + \delta\sigma_\varepsilon^2 \tag{10}$$

where

$$\delta = \frac{\text{VAR}(\theta_i)}{\text{VAR}(\varepsilon_i) + \text{VAR}\,(\theta_i)} = \frac{\sigma_\varepsilon^2 + k^2\sigma_\eta^2}{n\,\sigma_\varepsilon^2 + k^2\sigma_\eta{}^2} \tag{11}$$

and

$$E(\alpha|\tilde{S}_i) = t\,\tilde{S}_i + (1-t)\bar{\alpha} \tag{4'}$$

where $$t = \sigma_\alpha^2/(\sigma_\alpha^2 + \delta\sigma_\varepsilon{}^2) \tag{5'}$$

Replacing S_i by $\tilde{S}_i$, the problem of firm i given in (6) can be rewritten as

$$\max_{q_i} E(\Pi_i|\tilde{S}_i, \eta_i) = \max_{q_i} [(t\tilde{S}_i + (1-t)\bar{\alpha} - \eta_i - Q)q_i] \tag{12}$$

where (6) is now the special case in which $\delta = 1$, i.e. the case where either $\sigma_\varepsilon^2 \to 0$ in which case S_i contains sufficient information on α or $\sigma_\eta^2 \to \infty$ in which case S'_{-i} is an infinitely noisy signal and thus has no value.

Writing $Q = Q_{-i} + q_i$, the first order condition for maximising (12) with respect to q_i given the conjectural function (7), the combined signal (9), and η_i is

$$\frac{dE(\Pi_i)}{dq_i} = t\delta S_i + t(1-\delta)S'_{-i} + (1-t)\bar{\alpha} - Q_{-i} - \eta_i - (2 + c)q_i = 0 \qquad (13)$$

Substituting S'_{-i} fom (7) into (13) and solving for q_i in terms of S_i , η_i , Q_{-i} and $\bar{\alpha}$ yields the reaction function, incorporating reaction to information as

$$q_i = u + v(S_i - (Jv)^{-1}\eta_i) + wQ_{-i} \qquad (14)$$

where

$$u = \{(1-t)\bar{\alpha} - at(1-\delta)/b\}/J \qquad (15)$$

$$v = t\delta/J \qquad (16)$$

$$w = \{t(1-\delta)/b - 1\}/J \qquad (17)$$

and $$J = 2 + c + ct(1-\delta)/b \qquad (18)$$

A reaction function of the form (14) holds for any i^{th} firm. We must now investigate the joint response of all other firms, given their reaction functions (14), to an increase in one firm's output. Following Perry (1982) we not only take into account the direct effects but also the interaction among all the other firms (if firms i and j react to a change in the output of firm 1, then they also react to each other's reaction). Let the firm increasing its output be firm 1. For $i \neq 1$, we can write (14) as

$$q_i - wQ_{-i,\,1} = u + v(S_i - (Jv)^{-1}\,\eta_i) + wq_1 \qquad i = 2,\dots,n \tag{19}$$

where Q_{-i1} is the sum of outputs of all but the i^{th} and first firms. Summing (19) over i from 2 to n yields

$$Q_{-1} - (n-2)wQ_{-1} = (n-1)u + v\sum_{i=2}^{n}(S_i - (Jv)^{-1}\,\eta_i) + (n-1)wq_1 \tag{20}$$

and we can write

$$Q_{-1} = a + bS'_{-1} + cq_1$$

and more generally

$$Q_{-i} = a + bS'_{-i} + cq_i \tag{21}$$

where

$$k = (Jv)^{-1} \tag{22}$$

$$a = (n-1)u/M \tag{23}$$

$$b = (n-1)v/M \tag{24}$$

$$c = (n-1)w/M \tag{25}$$

and $$M = 1-(n-2)w \tag{26}$$

We have thus obtained a response function of the form of the conjectured response function (7). If the (a,b,c) defined as functions of (u,v,w) by equation (23), (24) and (25) are the same as those defining (u,v,w) in equations (15), (16) and (17), then a consistent conjectural variations equilibrium has been identified.

Note that a unique value of k greater than one exists to solve (22), since $(Jv)^{-1} \equiv (\sigma_\alpha^2 + \delta\sigma_\varepsilon^2)/\delta\sigma_\alpha^2$ from (5') and (16). Thus $(Jv)^{-1}$ is decreasing in δ. But, from (11), δ is increasing in k for $k \geq 0$. Thus $(Jv)^{-1}$ is decreasing in k. Further $(Jv)^{-1} > 1$ when $k = 1$ and is bounded at $1 + \sigma_\varepsilon^2/\sigma_\alpha^2$ as $k \to \infty$. Therefore (22) has a unique positive solution and we will simply denote this as k.

It remains to solve the six equations (15), (16), (17), (23), (24) and (25) for the six parameters a, b, c, u, v, w, in terms of the underlying parameters σ_α^2, σ_ε^2, σ_η^2, n, and $\bar{\alpha}$.

Taking the ratio of (16) to (17) and equating it to the ratio of (22) to (23) yields

$$\frac{t\delta}{\frac{t(1-\delta)}{b} - 1} = \frac{b}{c} = \frac{v}{w} \tag{27}$$

Using the first part of (27) to obtain b in terms of c yields

$$b = t(1-\delta) - t\delta c \tag{28}$$

Substitution of (28) into (17) permits w to be expressed as a function of c :

$$w = \delta c/((2+c)(1-\delta-\delta c) + c(1-\delta)) \tag{29}$$

Then substitution of (29) into (25) and (26) yields a quadratic equation in c :

$$\delta c^2 - c(2-(n+2)\delta) - (2-(n+1)\delta) = 0 \qquad (30)$$

The two roots of (30) represent conjectural coefficients which are consistent. The roots are:

(i) $c = c^* = (2-(n+1)\delta)/\delta$

(ii) $c = -1$

In each case, substituting back into (29), (28) and (27) yields associated unique values for w, b and v respectively. Then taking the ratio of (15) to (16) and of (23) to (24) yields

$$\frac{a}{b} = \frac{u}{v} = \{(1-t)\bar{\alpha} - \frac{a}{b}(1-\delta)t\}/t\delta \qquad (31)$$

Given b , a and u can be solved uniquely from (31). Thus in each case defined by the two roots of equation (30) there exists a unique set of parameters (a, b, c, u, v, w) which yields a consistent conjectural equilibrium. Inserting the parameters as coefficients in (14) and (22) respectively yields:

<u>Case (i)</u>

$$c^* = (2-(n+1)\delta)/\delta$$

$$q_i = G^{-1}[(\delta n-1)\delta\bar{\alpha} + (\delta n-1)\delta t(S'_i - \bar{\alpha}) + (2-(n+1)\alpha)Q_{-i}] \qquad (32)$$

$$Q_{-i} = (\delta n-1)\bar{\alpha} + (\delta n-1)t(S'_{-i}-\bar{\alpha}) + \frac{2-(n+1)\delta}{\delta}\, q_i \qquad (33)$$

where $G = (n-1)\delta + (n-2)(2-(n+1)\delta)$

Case (ii)

$c = -1$

$$q_i = \bar{\alpha} + t(S_i' - \bar{\alpha}) - Q_{-i} \tag{34}$$

$$Q_{-i} = \bar{\alpha} + t(S_{-i}' - \bar{\alpha}) - q_i \tag{35}$$

In the next section, each of the two cases are analysed in turn by considering the properties of the conjectural parameter and the implied equilibrium.

III. RESULTS

The two consistent conjectural variations identified in Section 2 lead to distinct industry equilibria. We define an equilibrium as follows:

Definition

An n-tuple $(q_1, \ldots, q_n)$ is an equilibrium if, given information $(\tilde{S}_i, \eta_i)$ no firm i wishes to change q_i, where $S'_{-i} = \frac{1}{b}(Q_{-i}-a-cq_i)$ and (a, b, c) form a consistent conjecture.

It is worth repeating that although the present model is a non-cooperative game, communications take place. Firms are assumed to announce their plans and it is from these that information is obtained about Q_{-i} and hence S'_{-i}.

It is assumed that no production can take place without a prior announcement so that any equilibrium is based on full information concerning competitors' supply plans.

The number n represents the number of firms in the industry, announcing plans and providing information, but not necessarily the number with positive outputs in an equilibrium, since firms are heterogeneous after receiving their signals and some may prefer to supply zero to the market. As the competitive conjecture is consistent even if there is no uncertainty about α, it could be anticipated that most of the interest of the extension to information transmission suggested here is centred on the other consistent conjecture, c^*. In the analysis of this case we will assume that variations in demand and costs are sufficiently limited for all n firms to produce at positive levels.

Proposition 1

(i) c^* is an increasing function of σ_α^2, a decreasing function of σ_η^2 and an ambiguous function of σ_ε^2 and n.

(ii) The following specific solutions exist in relation to σ_η^2.

	σ_η^2	k	δ	c^*
(a)	$\to 0$	$n + \sigma_\varepsilon^2/\sigma_\alpha^2$	$\frac{1}{n}$	$n-1$
(b)	$\dfrac{\sigma_\varepsilon^2}{\frac{n+1}{2} + \frac{\sigma_\varepsilon^2}{\sigma_\alpha^2}}$	$\frac{n+1}{2} + \frac{\sigma_\varepsilon^2}{\sigma_\alpha^2}$	$\frac{2}{n+1}$	0
(c)	$\to \frac{n}{(n-2)} \dfrac{\sigma_\varepsilon^2}{(\frac{n}{2} + \frac{\sigma_\varepsilon^2}{\sigma_\alpha^2})^2}$	$\frac{n}{2} + \frac{\sigma_\varepsilon^2}{\sigma_\alpha^2}$	$\frac{2}{n}$	-1

(iii) If $\sigma_\eta^2 < \frac{n}{n-2} k^{-2} \sigma_\varepsilon^2$ or equivalently $\delta < 2/n$ then $c^* \varepsilon (-1, n-1)$ exists.

(iv) c^* can take any value between -1 and $n-1$. More precisely, for any value $\hat{c} \varepsilon (-1, n-1)$, there exists positive finite n, σ_α^2, σ_ε^2, σ_η^2 such that $c^* = \hat{c}$.

Proof

The values of k and δ are simultaneously dependent on the parameters of the system. An explicit solution is not possible, but (16) and (22) yield, using (5') and (11):

$$\frac{1}{\delta} = \frac{n\sigma_\varepsilon^2 + k^2\sigma_\eta^2}{\sigma_\varepsilon^2 + k^2\sigma_\eta^2} = k - \frac{\sigma_\varepsilon^2}{\sigma_\alpha^2} \tag{36}$$

To show that $dc^*/d\sigma_\eta^2 < 0$, it is sufficient to show that $d\delta/d\sigma_\eta^2 > 0$. Note that

$$\frac{d\delta}{d\sigma_\eta^2} = \frac{\partial\delta}{\partial\sigma_\eta^2} + \frac{\partial\delta}{\partial k}\frac{dk}{d\sigma_\eta^2}$$

From (36), we have that

$$\frac{-1}{\delta^2}\frac{\partial\delta}{\partial k}dk - \frac{1}{\delta^2}\frac{\partial\delta}{\partial\sigma_\eta^2}d\sigma_\eta^2 = dk$$

and

$$\frac{dk}{d\sigma_\eta^2} = \frac{-\frac{1}{\delta^2}\frac{\partial\delta}{\partial\sigma_\eta^2}}{1 + \frac{1}{\delta^2}\frac{\partial\delta}{\partial k}}$$

so that

$$\frac{d\delta}{d\sigma_\eta^2} = \frac{\partial\delta}{d\sigma_\eta^2}\left\{\frac{1}{1 + \frac{1}{\delta^2}\frac{\partial\delta}{\partial k}}\right\}$$

Since, from (11), $\partial\delta/\partial\sigma_\eta^2 > 0$ and $\partial\delta/\partial k > 0$, δ is increasing and c^* decreasing in σ_η^2 .

The other comparative statics results in (i) can be shown in a similar way.

For part (ii), simply note that σ_η^2 , k , δ reported in the first three columns of the Table satisfy (36). Part (iii) is immediate from row (c) of the Table given that c^* is monotonic in σ_η^2 , as proved above. Part (iv) results from rows (a) and (c) of the Table and the continuity of c^* in σ_η^2 .

The comparative static results in Proposition 1 relate to output strategies incorporating information transmission. Thus more uncertainty over the prior predictor of α (σ_α^2) will lead to a higher value of c^* as firms put more reliance on market signals as a whole. Similarly as σ_η^2 increases, indicating less information content in output plans, so firms rely more on their own private signals and the coherence of the market is reduced. The effect of increased variance in the private signal (σ_ε^2) is two-fold; first it leads to more reliance on market signals relative to private signals, but secondly it leads to more reliance on the prior predictor so that other firms' outputs become less important for their information content concerning market demand and relatively more important as indicators of market supply. Similarly, an unamiguous effect from increasing n is not forthcoming.

Figures 1 and 2 show how c^* is influenced by σ_η^2 and σ_α^2 respectively. Note that if $n > 2$ then there is a sufficiently high noise level (sufficiently high σ_η^2) and a sufficiently low level of prior uncertainty over the market

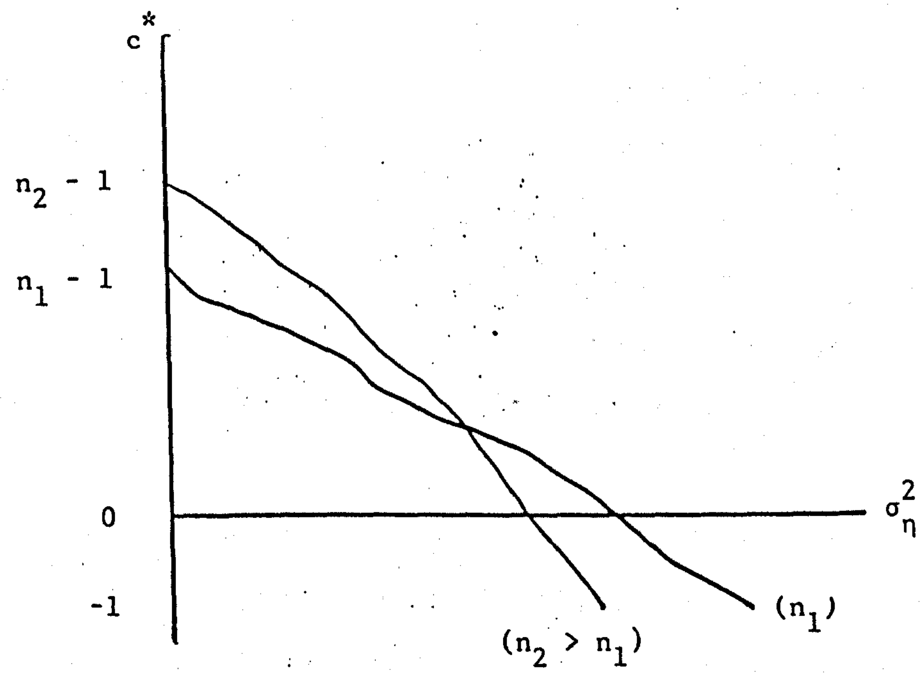

FIGURE 1 The Relationship between c^* and σ_η^2 .

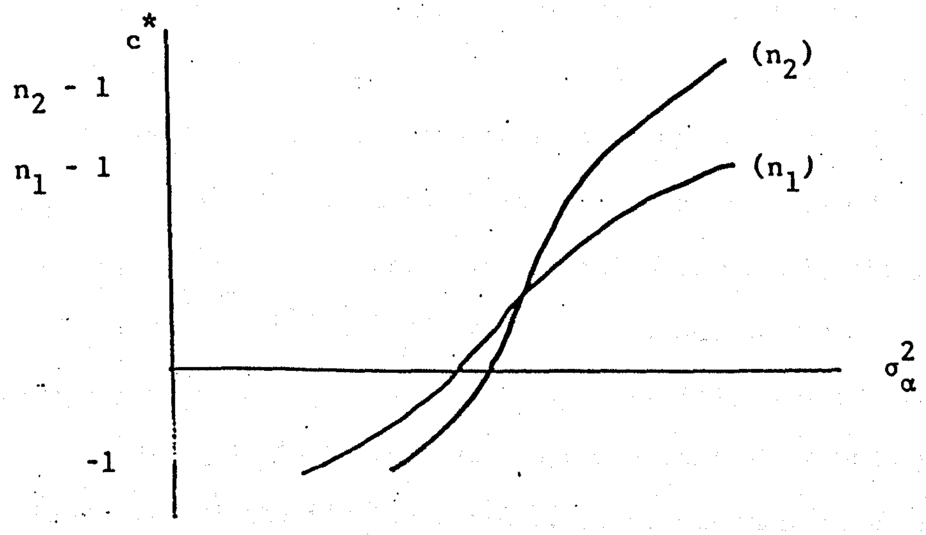

FIGURE 2 The Relationship between c^* and σ_α^2 .

size (σ_α^2 sufficiently low) that a consistent conjecture $C^* > -1$ does not exist. Also, note that increasing the number of firms reduces c^* if c^* is initially below some positive amount.

Figure 3 shows (for a set of numerical examples) how c^* is affected by the private signal variance σ_ε^2 . Note that c^* is little affected by quite large changes in σ_ε^2 away from extreme values, reflecting the confused effect of the greater need for information, as σ_ε^2 increased, being accompanied by a lower reliance by firms on their private signals and thus greater difficulty in extracting information.

The ambiguous effect of a change in n arise because when σ_η^2 is small it is as if firms pool their information. With pooled information, the variance of the total signal $\tilde{S}_i$ is decreasing in n implying that c^* is increasing in n when δ is close to $^1/n$.

The specific solution (ii. a) in Proposition 1 deserves some comments. In this case it is as if firms simultaneously collude over their choice of quantities $(c^* = n - 1)$ and pool their information $(\delta = {}^1/n)$. With σ_η^2 close to zero, Q_{-i} becomes a sufficient statistic for $\sum_{j\neq i} S_j$ and the apparent pooling of information follows using theorem 1 in McKelvey and Page (1986). The apparent collusion occurs since, with less noise in the signal S_i' , firms are more heartened/disheartened by the observed Q_{-i} and thus, as σ_η^2 gets smaller, move more and more in unison. In this special case information is imparted fully and involuntarily through announced plans. This is in contrast to the models of voluntary information sharing (see e.g. Clarke (1983)) where firms do not want to share information unless they may cooperate over strategies.

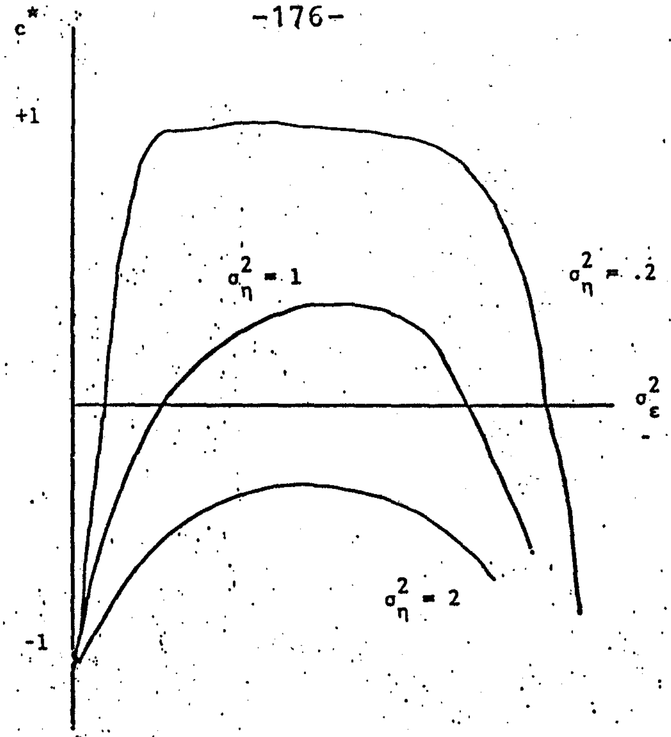

FIGURE 3.a The relation between c^* and σ_ε^2 for $n = 2$, $\sigma_\alpha^2 = 20$ and $\sigma_\eta^2 = .2, 1, 2$

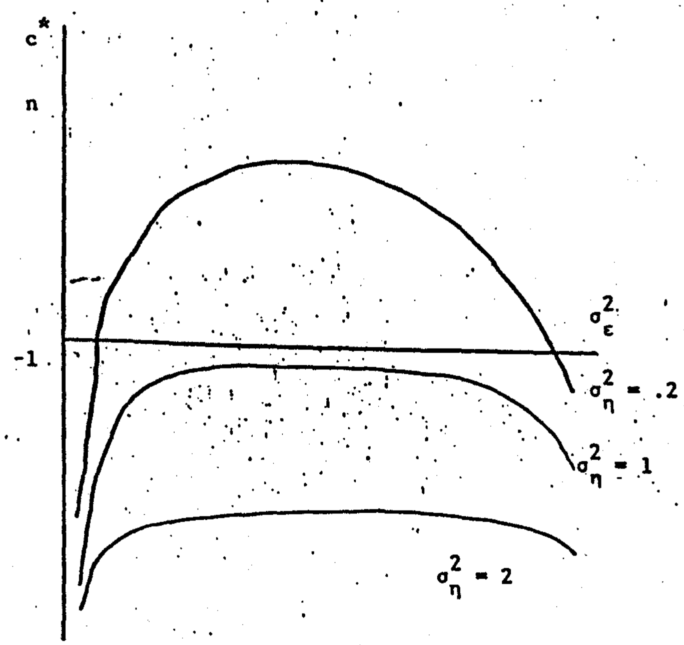

FIGURE 3.b The relation between c^* and σ_ε^2 for $n = 10$, $\sigma_\alpha^2 = 20$ and $\sigma_\eta^2 = .2, 1, 2$

Proposition 2

In an equilibrium with consistent conjectural variation c^*, aggregate output Q, individual firm output q_i, and the expected profit $E(\Pi_i)$ of firm i before the signals are received, are respectively

$$\text{(i)} \qquad Q = \frac{n\delta}{2}\bar{\alpha} + \frac{\delta t}{2}\sum_{i=1}^{n}(S_i' - \bar{\alpha}) \qquad (37)$$

$$\text{(ii)} \qquad q_i = \frac{\delta}{2}\bar{\alpha} + A(S_i' - \bar{\alpha}) + \left(\frac{\delta t}{2} - A\right)\frac{1}{n}\sum_{j=1}^{n}(S_j' - \bar{\alpha}) \qquad (38)$$

$$\text{where} \qquad A = \frac{\delta t(\delta n-1)}{(n-1)(2-\delta n)} > 0 \quad \text{for} \quad 1/n < \delta < 2/n \qquad (39)$$

$$\text{(iii)} \qquad E(\Pi_i) = \frac{\delta(2-n\delta)}{4}\bar{\alpha}^2 + \frac{\delta t(2-n\delta t)}{4}\sigma_\alpha^2 - \left(\frac{\delta t}{2}\right)^2\sigma_\varepsilon^2$$

$$+ \left[\frac{\delta}{2(2-n\delta)}\right) - \frac{1}{4}]\,\sigma_\eta^2 \qquad (40)$$

providing all n firms produce at positive levels.

Proof: sum (32) over all i, note that $\sum_{i=1}^{n} Q_{-i} = (n-1)Q$ and solve for Q to obtain (i). To derive (ii), subtract q_j from q_i using (34) to obtain

$$q_i - q_j = \frac{(\delta n-1)\delta t}{G}[(S_i' - \bar{\alpha}) - (S_j' - \bar{\alpha})] - \frac{2-\delta(n+1)}{G}(q_i - q_j)$$

solving yields

$$q_i - q_j = A[(S_i' - \bar{\alpha}) - (S_j' - \bar{\alpha})] \qquad (41)$$

Summing over all j and dividing by n yields

$$q_i = A[(S_i' - \bar{\alpha}) - \frac{1}{n}\sum_{j=1}^{n}(S_j' - \bar{\alpha})] + Q/N$$

substituting for Q from (i) yields (ii)

To show (iii), note that the price-cost margin for the i^{th} firm is

$$p_i = \alpha - Q - \eta_i$$

Using Q from (i) above allows p_i to be written purely in terms of the random variables α, ε_i and η_i

$$p_i = \alpha - \frac{n\delta}{2}\bar{\alpha} - \frac{\delta t}{2}\sum_{i=1}^{n}(S_i' - \bar{\alpha}) - \eta_i \tag{42}$$

Then combining (38) and (42) and taking unconditional expectations gives

$$E(\Pi_i) = E\{(\alpha - \frac{n\delta}{2}\bar{\alpha} - \frac{\delta t}{2}\sum_{j=1}^{n}(S_j' - \bar{\alpha}) - \eta_i)$$

$$(\frac{\delta}{2}\bar{\alpha} + A(S_i' - \bar{\alpha}) + (\frac{\delta t}{2} - A)\frac{1}{n}\sum_{j=1}^{n}(S_j' - \bar{\alpha}))\}$$

$$= \frac{\delta}{4}(2-n\delta)\bar{\alpha}^2 + A\sigma_\alpha^2 - A\frac{\delta t}{2}(n\sigma_\alpha^2 + \sigma_\varepsilon^2 + \sigma_\eta^2/\delta^2t^2)$$

$$+ \frac{A}{\delta t}\sigma_\eta^2 + (\frac{\delta t}{2} - A)\sigma_\alpha^2$$

$$- \frac{\delta t}{2}(\frac{\delta t}{2} - A)(n\sigma_\alpha^2 + \sigma_\varepsilon^2 + \sigma_\eta^2/\delta^2t^2) + t\delta(\frac{\delta t}{2} - A)\frac{1}{n}\sigma_\eta^2/\delta t$$

Rearranging yields (40).

To see why the assumption that all n firms produce at a positive level is necessary, note that

$$\delta \to \frac{2}{n} \Rightarrow A \to \infty \quad \text{and} \quad q_i \to \begin{cases} \infty & \text{if } (S_i' - \bar{\alpha}) > \mu \\ \frac{\bar{\alpha}}{n} + \frac{t}{n}\mu & \text{if } (S_i' - \bar{\alpha}) = \mu \\ -\infty & \text{if } (S_i' - \bar{\alpha}) < \mu \end{cases}$$

where $$\mu = \sum_{j=1}^{n} (S_j' - \bar{\alpha})/n$$

and from Proposition 1(ii.c)$\delta \to 2/n \Rightarrow c^* \to -1$. Thus as we approach the competitive conjecture, the output levels given in (38) are not bounded. As $\delta \to 2/n$ only the firm receiving the most encouraging signals will produce and supply the whole market. We will return to this question at the end of this section when considering the competitive conjecture.

The equilibrium associated with the c^* conjecture described in Proposition 2 is a function of all the information in the supplying industry. As we have assumed $q_i > 0 \ \forall i$, $E(\Pi_i) > 0 \quad \forall i$ is also implied in the absence of fixed costs. Before considering the comparative statics of the equilibrium it is useful to speculate how it could be achieved, given the information transmission which has to occur. In Proposition 3 a simple adjustment process of firms' plans is shown to lead to the equilibrium plans and outputs whatever the initial announcements.

<u>Proposition 3</u>.

From any initial announced positive output plans, there exists a continuous adjustment process whereby the industry reaches the equilibrium defined by Proposition 2.

Proof. Consider that firms adjust their planned output levels towards that indicated as ideal by their reaction function. That is:

$$\dot{q} = (u + vS_i' + wQ_{-i}) - q_i \qquad \forall i \tag{43}$$

where $\cdot$ implies differential with respect to time.

Now write these equations in vector form as

$$\dot{q} = \ell + Wq \tag{44}$$

where the typical (i^{th}) element of $\dot{q}$ is dq_i/dt , that of ℓ is $u + vS_i'$, that of q is q_i , and W is an $n \times n$ matrix with a principal diagonal of elements equal to -1 and all off-diagonal elements equal to w . The only eigenvalues 7/ of W are $-(1 + w)$ and $wn - (1+w)$. Since

$$w = \frac{2-\delta(n+1)}{G} = \frac{2-\delta(n+1)}{(n-1)\delta + (n-2)(2-(n+1)\delta)}$$

the eigenvalues are

$$-\frac{(n-1)(2-n\delta)}{G}$$

and $\dfrac{2(1-\delta n)}{G}$

respectively. Since $\frac{1}{n} < \delta < \frac{2}{n}$, $G > 0$, $(1-\delta n) < 0$ and $(2-n\delta) > 0$. Thus both eigenvalues are negative and the system converges to the equilibrium stated in Proposition 2.

Although other dynamic adjustment processes may not be stable, cases of instability may also occur in situations without information transmission.

There appears to be no particular added difficulty of reaching an equilibrium in the present analysis.

Proposition 4

Ex ante of any signals being received, the c^* conjecture yields an equilibrium with the following characteristics

(i) The expected industry output is $E(Q) = \frac{n\delta\bar{\alpha}}{2}$ and is increasing in n , $\bar{\alpha}$. and σ_η^2 , and decreasing in σ_α^2 .

(ii) The expected output per firm is $E(q) = \delta\alpha/2$ and is increasing in $\bar{\alpha}$ and σ_η^2 and decreasing in n and σ_α^2 .

(iii) The expected price-cost margin is $E(p) = \bar{\alpha}(1 - n\delta/2)$ and is increasing in $\bar{\alpha}$ and σ_α^2 and decreasing in n and σ_η^2 .

Proof. $E(Q)$, $E(q)$ and $E(p)$ are found by taking expectations of (37), (38) and (42) respectively. Differentiation, using (36) yields the comparative static results.

Proposition 4 shows that the average effect of greater σ_α^2 or smaller σ_η^2 is to move the industry towards a more coherent structure with associated lower industry supply and higher price-cost margins.

Although the c^* conjecture and its associated equilibrium is the main focus of our analysis, the competitive consistent conjecture is also of interest. However in this case, our assumption of always-active firms which removed some technical complications in our discussion of the c^* conjectural equilibrium, is no longer appropriate. We can in fact state the following proposition.

Proposition 5.

For the consistent conjecture $c = -1$, the following is almost always true

(i) Only one firm supplies the market

(ii) This firm is the one which has received the most encouraging signal, $S_i' = \alpha + \varepsilon_i - k\eta_i$, where $k = (t\delta)^{-1}$

(iii) This firm will produce just sufficient to deter the firm, which receives the second most encouraging signal, from producing.

(iv) If the number of potential supplying firms is finite the expected profit of any firm prior to receiving the informational signals is positive in the absence of fixed costs.

Proof. Part (i) For any assumed industry output $Q > q_i$, the i^{th} firm expects product price to be independent of its own output. Given its information it expects product price net of its revealed constant marginal costs to be:

$$P_i^e \equiv E(P_i | \tilde{S}_i, \eta_i) = t(\delta S_i + (1-\delta)S'_{-i}) + (1-t)\bar{\alpha} - \eta_i - Q$$

$$= t(\delta(S_i') + (1-\delta)S'_{-i}) + (1-t)\bar{\alpha} - Q \qquad (45)$$

as $k = (t\delta)^{-1}$. No equilibrium can exist with $P_i^e > 0$ unless all output is produced by firm i since firm i conjectures that all other firms will make room for any expansion in output. This is almost always true as the probability of two firms observing the same composite signal, $S_i' = S_i - k\eta_i$, leading to the possibility of sharing the market, is zero.

Further, any announcement of an output level which will supply the whole market is credible. If firm i is not the receiver of the most encouraging composite signal, then its announced output level will be such that there exists a firm j for which $p_j^e > 0$.

Part (ii). Using (45) we can write the difference between the i^{th} and j^{th} firm's net price expectation, given their respective information signals, as:

$$p_i^e - p_j^e = t\,(\delta(S_i' - S_j') + (1-\delta)(S_{-i}' - S_{-j}'))$$

$$= t(\delta - \frac{1-\delta}{n-1})\,(S_i' - S_j') \qquad (46)$$

As $\delta \geq \frac{1}{n}$, $p_i^e \gtrless p_j^e$ as $S_i' \gtrless S_j'$.

Part (iii). If firm i increases its output until all other firms have reduced their output to zero and if $p_i^e > 0$, then firm i is in equilibrium. For other firms to be in equilibrium, we must have $p_j^e \leq 0 \;\forall\; j \neq i$.

Part (iv). Ex ante of the signals being received, firms are identical. Thus they have the chance of being the sole supplier to the market. As $p_i^e > 0$ for the sole supplier, it makes positive profit while the profit of the other firms are zero in the absence of fixed costs. Hence the expected profit ex ante of the signals is positive.

Note the similarity between the firms' problem in this case and that faced by bidders in an open ascending bid (English) auction when bidding for an indivisible good with a common value. Such auctions, where bidders are uncertain about the value estimates but can infer other bidders' information from their bidding behaviour, have been studied by Milgrom and Weber (1982).

Secondly note that the firm producing is the most encouraged by its own composite signal and that this need not be the lowest cost firm. A two-firm case is shown in Figure 4 below, where although firm 2 has the lower costs, firm 1 will produce in equilibrium.

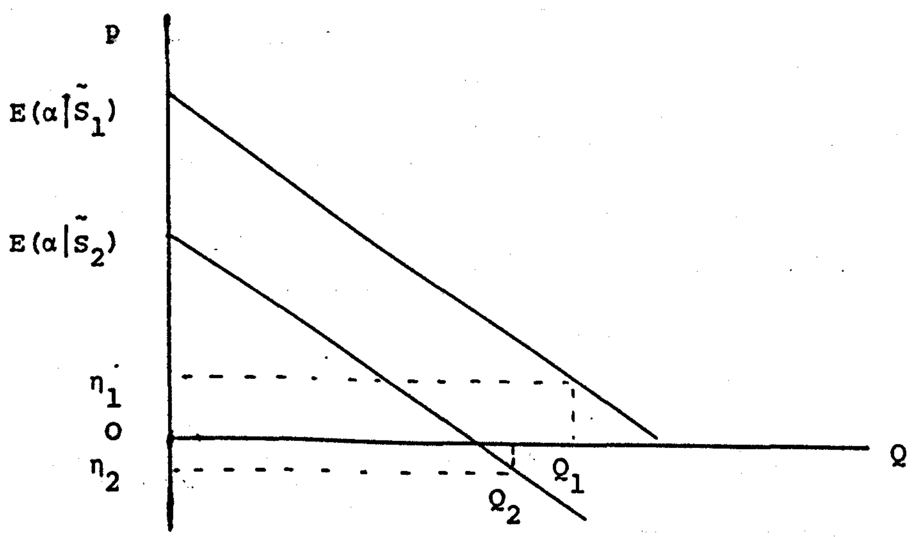

FIGURE 4 : The "Winning" Firm with Competitive Conjectures.

This leads to a third point. If firm 1 announces a high output, some firms may make no announcement and firm 1 may never obtain the information which could be collected from their participation. In an extreme case if firm 1 announces that it intends to produce Q_1 , it finds that it has all the market. If the monopoly output level is less than Q_1 , firm 1 will announce reductions in output until firm 2 enters the market. Firm 1 will then increase its output to slightly above Q_2, and will be in equilibrium. However this equilibrium will only reflect the signals S_1' and S_2' and not other firms' information (except insofar as they announce zero supply). A different starting point may yield more precise information from a larger set of interested firms and thus a different equilibrium since both Q_1 and Q_2 are likely to be affected.

With the c^* conjecture, the same kind of problems may occur when c^* is sufficiently near -1. Obviously, the bigger the variances of costs the more potentially heterogeneous the firms and the nearer c^* is to -1. These two factors combine to make the assumption that all firms produce positive outputs less appropriate when only very noisy signals are transmitted. Although the essential character of the analysis would not be much affected if some firms produced zero output in equilibrium, the study of c^* conjectures near to -1 would obviously benefit from a reformulation of the model, particularly the introduction of increasing marginal costs. As our interest in this paper has been to present a justification for positive consistent conjectures which occur where signals are relatively unnoisy and costs fairly similar, we have retained the simplicity of the constant cost model.

IV. CONCLUSION

An explanation for the cohesiveness of market supply has been advanced which is based on the transmission of information via quantity plans. Firms' conjectural variations take account of the signal which will be extracted by the rest of the industry from raising their intended supplies. Such conjectural variations can be positive and still consistent with observed responses in any experiment. The concept of consistent conjectural variations has thus been extended to situations where the competitive conjecture is inappropriate.

The conjecture c^* is determined by the underlying parameters of the model (see Proposition 1). It increases as a firm's prior estimate becomes less reliable (σ_α^2 increases) and as information transmission becomes less noisy (σ_η^2 decreases). Thus as the information content of quantity signals become more needed and more efficient, so the industry becomes more collusive in character, and (from Proposition 4) expected industry output decreases and expected price-cost margins increase.

Many assumptions, such as linear demand, homogeneous products and constant costs, contributed to the tractability of the analysis. Of more significance was the assumption that all firms in the industry contributed positive output in equilibrium with the c^* conjecture. As the competitive conjecture led to only one firm supplying the market (Proposition 5), it would seem likely that as c^* became near -1 so this assumption would become less tenable. However, no such problems are likely to occur when the parameters of the model determine c^* at positive levels. Finally, it has been assumed that only current announced plans can be put into effect.

Thus firms cannot announce a plan to produce one level of output while actually producing another.

The model outlined in this paper could be extended to relax a number of these assumptions in order to apply the underlying thesis to a wide selection of problems. Possible examples might include an extension to monopolistic competition where conjectural variations are a determinant of whether too many or too few products are produced in a Chamberlinian equilibrium (see Koenker and Perry, 1981, and Ireland, 1983). Also conjectural variations have been proposed (under a number of names) as a determinant of individual labour supply within a workers' cooperative; perhaps information transmission concerning rewards for work or required standards could be used to explain the level of conjectural responses to variations in individuals' labour supply. Further extensions relate to the inclusion of risk averseness of firms, where information transmission would have the secondary effect of reducing risk. It is hoped to explore these extensions in future research.

FOOTNOTES

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1. The reason for focussing attention on a one-stage game is its usefulness as a benchmark. With imperfect information, repeated games give rise to additional issues such as the formation of reputations, making the notion of information transfer more complicated, and to some extent obscuring the interpretation of results.

2. Consistent conjectures are discussed in Boyer and Moreaux (1983), Kalai and Stanford (1985), Kamien and Schwartz (1983), Perry (1982) and Ulph (1983). The use of conjectural variations has a long history dating from Frisch (1933).

3. One might even reject the notion of a conjectural variation in a one-stage game with simultaneous strategy choices. As argued in Daughety (1985), if a chosen strategy is irrevocable, it cannot react to the revelation of other players' strategies. This would be the case in a model without communication of any kind prior to the strategy choice. Firms would then hold point-certainty beliefs about the strategy choice of the other firms and optimise accordingly. Thus it is hardly surprising that the only equilibrium is of the Cournot type. For an extensive treatment, see Daughety (1985).

4. An alternative scenario would rely on the infeasibility of any firm announcing a false plan. Then (i) firms obtain their private information; (ii) they announce what they truthfully plan to produce; (iii) firms infer additional information from the truthful plans of others; (iv) based on their augmented information set, firms simultaneously choose their output levels. Both scenarios yield the same outcome but are predicated on different assumptions. That in the text relies on the assumptions that firms cannot deviate from final plans but that no plan is final until all firms are in equilibrium. The alternative outlined above relies on plans always being true representations of firms' intentions.

5. The "Tendency Survey for Manufacturing Industries" in Denmark is fairly typical. Results are calculated from about 700 major manufacturing enterprises representing 60% of total manufacturing employment, and relate to some 70 market groups.

6. Equation (4) is true not only for normal errors but also for a range of other distributions. See Ericson (1969).

7. Note that an n-square matrix with all elements equal to $1/n$ is idempotent and thus has eigenvalues of 0 and 1 only. Thus a matrix with all elements equal to w has eigenvalues equal to 0 and nw only and so the eigenvalues of W are these values minus $(1+w)$.

CHAPTER VII.

General Conclusion

Some general conclusions to emerge are firstly that the effect of private information and information transmission in the context of oligoipoly models is ambiguous. This, to some extend negative, result implies that one should be careful when using the results of these models to draw policy conclusions. This is clear from the analysis in chapter II and III, where we showed that the commment by Clarke(1983b), that information sharing is only preferred by firms if they can also collude over strategies, and therefore information sharing is proof of collusion, is not generally true. This potential reversal of the results of the literature also indicate that the results of the models are non-robust to changes in the functional forms.

Secondly, models which under certainty are relatively simple, can get quite complicated once uncertainty and private information is introduced (as can be seen from e.g. chapters IV and V). Further, these models which allows for private information also allows a broader range of results as shown in chapter VI. One is then left with a feeling that by careful choice of functional forms, information structures and stochastic specifications, almost any result could be shown.

This suggests that models should be selected on the basis of empirical analysis. The final conclusion to emerge from this study is that, although this may be very true, taking uncertainty and private information as a datum implies that equations may have to be estimated by non-standard methods and that non-standard tests may have to be used. It certainly gives yet more reasons for testing the assumptions underlying the estimation procedure and tests. This is most clearly brought out by the analysis in chapters IV and V. Here under certainty the model mimicks the Cournot oligopoly model. Once uncertainty is introduced this is no longer true. We find that market shares not only are random but also that the market share of different firms in the same industry may follow different distributions. This have serious implications if we want to use market shares as explanatory variables in econometric models, because ordinary least squares need no longer be unbiased and efficient. Thus it is necessary to convince oneself that the explanatory variables are deterministic and to test the implied assumptions regarding the error term. Further if we take the existence of mixed strategies serious, and if we want to explain price-cost margins, then these will almost surely not follow a normal distribution. This implies that standard tests are at most asymptotically valid and hence that tests for normality are essential. Finally the functional forms of the equations to be estimated should be based on models where the likely sources of uncertainty are taken explicitely into consideration, rather than based on deterministic models, and estimators consistent with the derived stochastic structure (e.g. maximum likelihood) should be used.

BIBLIOGRAPHY.

Allen, B. and M. Hellwig, 1986, "Bertrand-Edgeworth Oligopoly in Large Markets", Review of Eonomic Studies, vol. 53, pp. 175-204.

Andersen, T.M., 1985, "Recent Developments in the Theory of Efficient Capital Markets", Kredit und Kapital, vol. 18, pp. 347-371.

Archibald, G.C., B.C. Eaton and R.G. Lipsey, 1986, "Address Models of Value Theory" in J.E. Stiglitz and G.F. Mathewson (Eds.), New Developments in the Analysis of Market Structure, MacMillan.

Başar, T. and Y.-C. Ho, 1974, "Informational Properties of the Nash Solutions on Two Stochastic Nonzero-Sum Games", Journal of Economic Theory, vol. 7, pp. 370-387.

Boyer, M. and M. Moreaux, 1983, "Conjectures, Rationality and Duopoly Theory", International Journal of Industrial Organization, vol. 1, pp. 23-41.

Boyer, M. and M. Moreaux, 1983, "Being a Leader or a Follower: Reflections on von Stackelberg", International Journal of Industrial Organization vol. 5, pp. 175-192.

Bresnahan, T.F., 1981, " Duopoly Models with Consistent Conjectures", American Economic Review, vol. 71, pp. 934-945.

Clarke, R.N., 1982, "Endogenous Mergers in Stochastic Cournot-Nash Oligopoly", SSRI working paper 8230, University of Wisconsin-Madison.

Clarke, R.N., 1983a, "Duopolists don't wish to Share Information", Economics Letters, vol. 11, pp. 33-36.

Clarke, R.N., 1983b, "Collusion and the Incentive for Information Sharing", Bell Journal of Economics, vol. 14, pp. 383-394.

Cyert, R.M. and M.H. DeGroot, 1974, "Rational Expectations and Bayesian Analysis", Journal of Political Economy, vol. 82, pp. 521-536.

Dasgupta, P. and E. Maskin, 1986a, "The Existence of Equilibrium in Discontinuous Economic Games I: Theory", Review of Eonomic Studies, vol. 53, pp. 1-26.

Dasgupta, P. and E. Maskin, 1986a, "The Existence of Equilibrium in Discontinuous Economic Games II: Applications", Review of Eonomic Studies, vol. 53, pp. 27-42.

d'Aspremont, C., A. Jacquemin, J.J. Gabszewicz and J.A. Weymark, 1983, "On the Stability of Collusive Price Leadership", Canadian Journal of Economics, vol. 16, pp. 17-25.

Daughety, A.F., 1985, "Reconsidering Cournot: the Cournot equilibrium is consistent", Rand Journal of Economics, vol. 16, pp. 368-379.

Davidson, C. and R. Deneckere, 1986, "Long-run competition in capacity, short-run competition in price, and the Cournot model", Rand Journal of Economics, vol. 17, pp. 404-415.

Dixon, H., 1984,"The Existence of Mixed-Strategy Equilibria in a Price-Setting Oligopoly with Convex Costs", Economic Letters, vol. 16, pp. 205-212.

Dixon, H.,1985, "Strategic Investment with Competitive Product Markets", Journal of Industrial Economics vol 33, pp. 483-500.

Dixon, H., 1986a, "Strategic Investment with Consistent Conjectures", Oxford Economic Papers, vol 38 supplement, pp. 111-128.

Dixon, H., 1986b, "The Cournot and Bertrand Outcomes as Equilibria in a Strategic Metagame", Economic Journal vol. 96 supplement, pp. 59-70.

Dixon, H., 1987, "The General Theory og Household and Market Contingent Demand", The Manchester School vol. 55, pp. 287-304.

Donsimoni, M.-P., 1985, "Stable Heterogenous Cartels", International Journal of Industrial Organization, vol. 3, pp. 451-468.

Dovrick, S., 1986, "von Stackelberg and Cournot Doupoly: Choosing Roles", Rand Journal of Economics, vol. 17, pp. 251-260.

Eaton, J. and H. Kierzkowski, 1984, "Oligopolistic Competition, Product Variety, Entry Deterrence, and Technology Transfer", Rand Journal of Economics, vol. 15, pp. 99-107.

Edgeworth, F.Y., 1925, " The Pure Theory of Monopoly", in F.Y. Edgeworth, Papers relating to political Economy,, vol.1 London, pp. 111-142.

Ericson, W.A., 1969, "A Note on the Posterior Mean of a Population Mean", Journal of the Royal Statistical Society, vol. 31, pp. 332-334.

Frich, R., 1933, "Monopole-Polypole - la notion de force dans l'economie", Nationaløkonomisk Tidsskrift, vol. 71, pp. 241-259.

Gal-Or, E., 1985a, "Information Sharing by Oligopolists" Econometrica, vol. 53, pp. 329-344.

Gal-Or, E., 1985b, "First Mover and Second Mover Advantages", International Economic Review, vol. 26, pp. 649-653.

Gal-Or, E., 1986, "Information Transmission - Cournot and Bertrand Equilibrium", Review of Economic Studies, vol. 53, pp. 85-92.

Gal-Or, E., 1987, "First Mover Disadvantage with Private Information", Review of Economic Studies vol. 54, pp. 279-292.

Gelman, J.R. and S.C. Salop, 1983, Judo Economics: Capacity Limitation and Coupon Competition", Bell Journal of Economics, vol. 14, pp. 315-325.

Grossman, S.J., 1981, "An Introduction to the Theory of Rational Expectations Under Asymmetric Information", Review of Economic Studies, vol. 48, pp. 541-559.

Harris, R. and T.R. Lewis, 1982, "Strategic Commitment Under Uncertainty with Private Information", Department of Economics Discussion Paper no. 82-24, University of British Columbia.

Ireland, N.J., 1986, "Price Leadership and Product Innovation in a 'Natural' Dominant Firm Market", Forthcomming in European Journal of Political Economy.

Ishii, Y., 1977, "On the Theory of the Competitive Firm under Demand Uncertainty: note", American Economic Review vol. 67, pp. 768-769.

Jaffe, J.F. and R.L. Winkler, 1976, "Optimal Speculation Against an Efficient Market", Journal of Finance, vol. 31, pp. 49-61.

Kamien, M.I. and N.L. Schwartz, 198 , "Conjectural Variations", Canadian Journal of Economics, vol. 16, pp. 191-210.

Kalai, E. and W. Stanford, 1985, "Conjectural Variations Strategies in Accelerated Cournot Games", International Journal of Industrial Organization, vol. 3, pp. 133-152.

Kirman, A.P., 1975, "Learning by Firms about Demand Conditions", In R.H. Day and T. Groves(Eds.), Adaptive Economic Models, Academic Press.

Kirman, A.P., 1983, "On Mistaken Beliefs and Resultant Equilibria", In R. Frydman and E.S. Phelps(Eds.), Individual Forecasting and Aggregate Outcomes, Cambridge University Press.

Koenker, R.W. and M.K. Perry, 1981, "Product Differentiation, Monopolistic Competition and Public Policy", Bell Journal of Economics, vol. 12, pp. 217-231.

Kreps, D.M. and J.A. Scheinkman, 1983, "Quantity Precommitment and Bertrand Competition Yield Cournot Outcomes", Bell Journal of Economics, vol. 14, pp. 326-337.

Levitan, R. and M. Shubik, 1972, "Price Duopoly and Capacity Constranits", International Economic Review, vol. 13, pp. 111-122.

Li, L., 1985, "Cournot Oligopoly with Information Sharing", Rand Journal of Economics, vol. 16, pp. 521-536.

McKelvey, R.D. and T. Page, 1986, "Common Knowledge, Consensus and Aggregare Information", Econometrica, vol. 54, pp. 109-127.

Milgrom, P.R. and R.J. Weber, 1982, "A Theory of Auktions and Competitive Bidding", Econometrica, vol. 50., pp. 1089-1122.

Nalebuff, B. and R. Zeckhauser, 1986, "The Ambiguous Antitrust Implications of Information Sharing", Discussion paper no. 106, Woodrow Wilson School, Princeton University.

Novshek, W., 1980,"Equilibrium in a Simple Spatial (or Differentiated Products) Model", Journal of Economic Theory, vol. 22, pp. 313-326.

Novshek, W. and H. Sonnenshein, 1982, "Fulfilled Expectations Cournot Duopoly with Information Acquisition and Release", Bell Journal of Economics, vol. 13, pp. 214-218.

Oi, W.Y., 1961, "The Desirability of Price Instability under Perfect Competition", Econometrica, vol. 29, pp. 58-64.

Okada, A., 1982, "Information Exchange between Duopolistic Firms", Journal of the Operations Research Society of Japan, vol. 25, pp. 58-75.

Osborne, M.J. and C. Pitchik, 1986, "Price Competition in a Capacity Constrained Duopoly", Journal of Economic Theory, vol. 38, pp. 238-260.

Perry, M.K., 1982, "Oligopoly and Consistent Conjectural Equilibria", Bell Journal of Economics, vol. 13, pp. 197-205.

Ponssard, J.-P., 1979, "The Strategic Role of Information on the Demand Function in an Oligopolistic Market", Management Science, vol. 25, pp. 243-250.

Riordan, M.H., 1985, "Imperfect Information and Dynamic Conjectural variations", Rand Journal of Economics, vol. 16, pp. 41-50.

Roberts, K., 1985, "Cartel Behaviour and Adverse Selection", in P.A. Geroski, L. Phlips and A. Ulph (Eds.), A Symposium on Oligopoly, Competition and Welfare, Journal of Industrial Economics, vol. 33, pp. 401-413.

Sakai, Y., 1984, "The Role of Information in a Stackelberg-Type Duopolistic Market", mimeo, University of Tsukuba, Japan.

Sakai, Y., 1985, "The value of Information in a Simple Duopoly Model", Journal of Economic Theory, vol. 36, pp. 36-54.

Salant, S.W., S. Switzer and R.J. Reynolds, 1983, "Losses from Horizontal Merger: The Effects of an Endogenous Change in Industry Structure on Cournot-Nash Equilibrium", Quarterly Journal of Economics, vol. 98, pp. 185-199.

Sandmo, A., 1971, "On the Theory of the Competitive Firm under Price Uncertainty", American Economic Review vol. 61, pp. 65-73.

Selten, R., 1973, "A simple model of Imperfect Competition, where 4 are Few and 6 are Many", International Journal of Game Theory, vol. 3, pp. 141-201.

Shapiro, C., 1986, "Exchange of Cost Information in Oligopoly", Review of Economic Studies, vol. 53, pp. 433-446.

von Stackelberg, H., 1934, Marktform und Gleichgewicht, verlag von Julius Springer, Wien und Berlin.

von Stackelberg, H., 1938, "Probleme der unvollkommenen Konkurrenz", Weltwirtschaftliches Archiv, vol. 48, pp. 95-141.

von Stackelberg, H., 1952, The Theory of the Market Economy, Translated by A.T. Peacock, London.

Ulph, D., 1983, "Rational Conjectures in the Theory of Oligopoly", International Journal of Industrial Organisation, vol. 1, pp. 131-154.

Varian, H.R., 1980, "A Model of Sales", American Economic Review, vol 70, pp. 651-659.

Vives, X., 1984, "Duopoly Information Equilibrium: Cournot and Bertrand", Journal of Economic Theory, vol. 34, pp. 71-94.

Vives, X., 1986, "Capacity pre-commitment, Technology Flexibility and Oligopoly Outcomes", International Journal of Industrial Organization, vol. 4, pp. 217-229.